

# Data-Driven Stabilization of Continuous-Time LTI Systems from Noisy Input–Output Data

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**Abstract**—We present an approach to compute stabilizing controllers for continuous-time linear time-invariant systems directly from an input–output trajectory affected by process and measurement noise. The proposed output-feedback design combines (i) an observer of a non-minimal realization of the plant and (ii) a feedback law obtained from a linear matrix inequality (LMI) that depends solely on the available data. Under a suitable interval excitation condition and knowledge of a noise energy bound, the feasibility of the LMI is shown to be necessary and sufficient for stabilizing all non-minimal realizations consistent with the data. We further provide a condition for the feasibility of the LMI related to the signal-to-noise ratio, guidelines to compute the noise energy bound, and numerical simulations that illustrate the approach.

## I. INTRODUCTION

Recent trends in automatic control highlight a growing reliance on data-based methods. Among them, the design of controllers directly from data—rooted in classical adaptive control [1]—has gained significant momentum. In particular, *data-driven control* [2] has become a central paradigm, inspired by the wave of results related to Willems et al.’s fundamental lemma [3] and fueled by the increasing availability of large datasets. This approach aims to map the available data into controllers via linear matrix inequalities (LMIs) or related optimization programs, leading to an end-to-end methodology that avoids any modeling or identification step.

In the discrete-time domain, two fundamental contributions to data-driven control are [4] and [5], focusing on linear time-invariant (LTI) systems. These works proposed LMI formulations based respectively on data-based closed-loop system parameterizations and the data informativity framework. Later, data-driven LMIs particularly suited to noisy datasets have been developed from open-loop system parameterizations [2, 6, 7]. Notably, these approaches enable the enforcement of design specifications (e.g., closed-loop stability and  $\mathcal{H}_2/\mathcal{H}_\infty$  performance) for all systems consistent with the data. Further developments include approaches for linear quadratic regulation [8], partial model knowledge [9],

time-varying systems [10], nonlinear systems [11], systems subject to input saturations [12], and output-feedback control based on the behavioral framework [13] or non-minimal realizations constructed from shifted input–output data [14].

In the continuous-time setting, similar LMIs have been developed for data-driven state-feedback control [4, 15, 16], also accounting for the presence of noise, but at the price of requiring state derivative measurements in the dataset. To circumvent this limitation, research efforts have been dedicated to achieving *derivative-free* state-feedback approaches [17, 18]. Despite these advancements, data-driven control of continuous-time systems from input–output data remains far less developed. In this direction, [19] and [20] proposed a derivative-free framework to design output-feedback controllers with a filter of the input–output data acting as an observer of a non-minimal realization of the plant. While this approach achieves stabilization and output regulation in the noise-free setting, to the best of the authors’ knowledge, no method can yet handle noisy input–output data.

Motivated by this gap, this paper proposes an approach for data-driven stabilization of a class of multi-input multi-output (MIMO) systems subject to process and measurement noise that generalizes the results of [20]. The proposed control design comprises a filter of the input–output signals and a linear feedback law depending on the filter states. The gains of the feedback law are computed with an LMI obtained by postprocessing the offline data through the filter dynamics. As compared with [20], we leverage the quadratic matrix inequality (QMI) formalism of [7] to obtain a robust LMI formulation whose decision variables do not scale with the data, thus also ensuring application to large datasets. Under an interval excitation condition and assuming the knowledge of a noise energy bound, we use a matrix version of the S-lemma to prove that the feasibility of the LMI is necessary and sufficient for stabilizing all non-minimal realizations consistent with the data, characterized by a bounded matrix ellipsoid. We then relate the feasibility of the LMI to the signal-to-noise ratio of the dataset. Finally, we show a method to compute the noise energy based on assumptions on the noise and the tuning of the filters.

The paper is organized as follows. In Section II, we introduce the considered control problem. In Section III, we illustrate our approach and its theoretical guarantees. In Section IV, we provide a sufficient condition for LMI feasibility, while Section V shows how to compute the noise energy bound. Finally, Section VI presents two numerical examples and Section VII concludes the paper. Further details and the proofs can be found in [21].

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*Notation:* We use  $\mathbb{R}$  to denote the set of real numbers.  $I_j$  denotes the identity matrix of dimension  $j$ , while  $0_j$  and  $0_{j \times k}$  denote zero matrices of dimensions  $j \times j$  and  $j \times k$ .  $\otimes$  is the Kronecker product between matrices. We denote by  $\sigma(R)$  the spectrum of a square matrix  $R$ . Given a symmetric matrix  $S$ ,  $\lambda_{\min}(S)$  and  $\lambda_{\max}(S)$  denote its minimum and maximum eigenvalues, while  $S \succ 0$  (resp.,  $S \succeq 0$ ) denotes that it is positive definite (resp., positive semidefinite). Similarly,  $\prec 0$  and  $\preceq 0$  are used for negative definite and semidefinite matrices. For symmetric matrices  $S_1$  and  $S_2$ , we write  $S_1 \preceq S_2$  if  $S_1 - S_2 \preceq 0$ . Given a signal  $f(\cdot) : [0, T] \rightarrow \mathbb{R}^l$ , its finite-horizon  $\mathcal{L}_2[0, T]$  norm is  $\|f(\cdot)\|_{2, [0, T]} := \left( \int_0^T |f(\tau)|^2 d\tau \right)^{1/2}$ . Finally, the  $\mathcal{H}_\infty$  norm of a transfer function matrix  $\mathcal{G}(s)$  is denoted by  $\|\mathcal{G}\|_\infty$ .

## II. PROBLEM STATEMENT

We consider continuous-time linear time-invariant systems described by the following input–output model:

$$\mathcal{D}\left(\frac{d}{dt}\right)(y - v) = \mathcal{N}\left(\frac{d}{dt}\right)u + \mathcal{N}_w\left(\frac{d}{dt}\right)w, \quad (1)$$

where  $u \in \mathbb{R}^m$  is the control input,  $y \in \mathbb{R}^p$  is the measured output,  $w \in \mathbb{R}^q$  is the process noise, and  $v \in \mathbb{R}^p$  is the measurement noise, while, for some known positive integer  $n$  that we denote as *order* of the differential equation (1),

$$\begin{aligned} \mathcal{D}(\xi) &:= I_p \xi^n + A_{n-1} \xi^{n-1} + \dots + A_1 \xi + A_0 \\ \mathcal{N}(\xi) &:= B_{n-1} \xi^{n-1} + \dots + B_1 \xi + B_0 \\ \mathcal{N}_w(\xi) &:= E_{n-1} \xi^{n-1} + \dots + E_1 \xi + E_0 \end{aligned} \quad (2)$$

are polynomial matrices with unknown matrix coefficients  $A_i \in \mathbb{R}^{p \times p}$ ,  $B_i \in \mathbb{R}^{p \times m}$ ,  $i \in \{0, \dots, n-1\}$  and known matrix coefficients  $E_i \in \mathbb{R}^{p \times q}$ ,  $i \in \{0, \dots, n-1\}$ . Note that the solutions of the differential equation (1) are intended in the weak sense as per [22, Def. 2.3.7], so that the signals are not required to be differentiable but only locally integrable.

We represent the input–output model (1) with the following state-space realization, having state  $x \in \mathbb{R}^{np}$ :

$$\begin{aligned} \dot{x} &= \underbrace{\begin{bmatrix} 0_p & \cdots & 0_p & -A_0 \\ I_p & & & -A_1 \\ & & & \vdots \\ & & I_p & -A_{n-1} \end{bmatrix}}_A x + \underbrace{\begin{bmatrix} B_0 \\ B_1 \\ \vdots \\ B_{n-1} \end{bmatrix}}_B u + \underbrace{\begin{bmatrix} E_0 \\ E_1 \\ \vdots \\ E_{n-1} \end{bmatrix}}_E w \\ y &= \underbrace{\begin{bmatrix} 0_p & \cdots & 0_p & I_p \end{bmatrix}}_C x + v, \end{aligned} \quad (3)$$

where the pair  $(C, A)$  is observable. We also assume the following property.

**Assumption 1.** *The pair  $(A, B)$  is controllable.*

Suppose that an experiment is performed on system (3) over the time interval  $[0, T]$ , with  $T > 0$ . Specifically, given an unknown initial condition  $x(0) \in \mathbb{R}^{np}$ , an input trajectory  $u(\cdot) : [0, T] \rightarrow \mathbb{R}^m$ , and unknown noise signals  $w(\cdot) : [0, T] \rightarrow \mathbb{R}^q$ ,  $v(\cdot) : [0, T] \rightarrow \mathbb{R}^p$ , let the corresponding state and output trajectories be  $x(\cdot) : [0, T] \rightarrow \mathbb{R}^{np}$  and

$y(\cdot) : [0, T] \rightarrow \mathbb{R}^p$ , with  $x(\cdot)$  an absolutely continuous function. Then, suppose that the continuous-time dataset  $\{(t, u(t), y(t)) : t \in [0, T]\}$  is collected during the experiment. Our goal is to use such a dataset to design a stabilizing controller for system (1) (equivalently, system (3)) without any prior knowledge of  $A_i, B_i, i \in \{1, \dots, n-1\}$  except for the order  $n$ . Namely, we aim to compute the matrices  $A_c, B_c, C_c$ , and  $D_c$  of a dynamic controller of the form

$$\begin{aligned} \dot{x}_c &= A_c x_c + B_c y \\ u &= C_c x_c + D_c y, \end{aligned} \quad (4)$$

with controller state  $x_c \in \mathbb{R}^\mu$ , so that the following matrix, obtained from the interconnection of (3) and (4), is Hurwitz:

$$\begin{bmatrix} A + BD_c C & BC_c \\ B_c C & A_c \end{bmatrix}. \quad (5)$$

## III. DATA-DRIVEN STABILIZATION FROM NOISY INPUT–OUTPUT DATA

We now illustrate the proposed data-driven stabilization approach, whose implementation is provided in Algorithm 1. The central idea is to build a non-minimal realization of the plant (Section III-A) that has a structure amenable to observer design. This allows us to split the dynamic controller into an observer of the non-minimal realization and a feedback law depending on the observer states (Section III-B). In particular, the feedback gain is designed to quadratically stabilize all realizations consistent with the data (Section III-C) by employing a data-based LMI derived from the matrix S-lemma [7, Thm. 4.10] (Section III-D).

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### Algorithm 1 Data-Driven Stabilization from Noisy I–O Data

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#### Initialization

*Measured dataset:*

$$\{(t, u(t), y(t)) : t \in [0, T]\}. \quad (6)$$

*Controller state dimension:*  $\mu := n(p + m)$ .

*Tuning:*  $\Lambda \in \mathbb{R}^{n \times n}$  Hurwitz with  $n$  distinct eigenvalues;  $\Gamma \in \mathbb{R}^n$  such that  $(\Lambda, \Gamma)$  is controllable;  $\Delta \in \mathbb{R}^{p \times p}$ ,  $\Delta = \Delta^\top \succeq 0$ . Filter matrices:

$$F := I_{p+m} \otimes \Lambda, \quad G := \begin{bmatrix} 0_{np \times m} \\ I_m \otimes \Gamma \end{bmatrix}, \quad L := \begin{bmatrix} I_p \otimes \Gamma \\ 0_{nm \times p} \end{bmatrix}. \quad (7)$$

#### Filtering

*Filter signals:* using (6), compute for all  $t \in [0, T]$ :

$$\zeta(t) := \begin{bmatrix} \chi(t) \\ \hat{z}(t) \end{bmatrix} := \begin{bmatrix} e^{\Lambda t} \Gamma \\ \int_0^t e^{F(t-\tau)} (Gu(\tau) + Ly(\tau)) d\tau \end{bmatrix}. \quad (8)$$

#### Controller Design

*LMI:* find  $P \in \mathbb{R}^{\mu \times \mu}$  and  $Q \in \mathbb{R}^{m \times m}$  satisfying the linear matrix inequality (S) reported in the next page.

*Control gain:*

$$K := QP^{-1}. \quad (9)$$

*Controller matrices:* in (4), let:

$$A_c = F + GK, \quad B_c = L, \quad C_c = K, \quad D_c = 0_{m \times p}. \quad (10)$$


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### A. Non-Minimal Realization for Data-Driven Control

Inspired by classical results in adaptive control [23, Ch. 4] and following the state-space framework developed in [20], we now introduce a non-minimal realization of the transfer function matrix  $\mathcal{D}^{-1}(s)\mathcal{N}(s)$ , characterizing the noise-free input–output behavior of (1). Let  $(\Lambda, \Gamma)$  be a controllable pair as described in Algorithm 1, and let

$$\mathcal{D}_\lambda(\xi) := \xi^n + \lambda_{n-1}\xi^{n-1} + \dots + \lambda_1\xi + \lambda_0 \quad (11)$$

be the characteristic polynomial of  $\Lambda$ , having  $n$  distinct roots with negative real part. Then, define  $F \in \mathbb{R}^{\mu \times \mu}$ ,  $G \in \mathbb{R}^{\mu \times m}$ ,  $L \in \mathbb{R}^{\mu \times p}$  as in (7), with  $\mu := n(p+m)$  that represents, as shown below, the dimension of the non-minimal realization.

The following result will be used throughout the paper.

**Lemma 1.** *Under Assumption 1 and given  $F$ ,  $G$ , and  $L$  as in (7), there exist full-rank matrices  $\Pi \in \mathbb{R}^{np \times \mu}$ ,  $H \in \mathbb{R}^{p \times \mu}$  that satisfy*

$$\Pi(F + LH) = A\Pi, \quad \Pi G = B, \quad H = C\Pi, \quad (12)$$

and, in addition, ensure the following properties:

- 1)  $(F + LH, G)$  is controllable.
- 2)  $A - \Pi LC$  has the form

$$\tilde{\Lambda} := A - \Pi LC = \left[ \begin{array}{ccc|c} 0_p & \cdots & 0_p & -\lambda_0 I_p \\ I_p & & & -\lambda_1 I_p \\ & \ddots & & \vdots \\ & & I_p & -\lambda_{n-1} I_p \end{array} \right]. \quad (13)$$

Using  $\Pi$  and  $H$  from Lemma 1, we obtain from [20, Lemma 1] that the following dynamical system

$$\begin{aligned} \dot{z} &= (F + LH)z + Gu \\ y &= Hz, \end{aligned} \quad (14)$$

with  $z \in \mathbb{R}^\mu$ , is a controllable and detectable (but not observable) non-minimal realization of  $\mathcal{D}^{-1}(s)\mathcal{N}(s)$ . In particular, the controllable and observable subsystem of (14), parameterized via  $x = \Pi z$ , obeys  $\dot{x} = Ax + Bu$ ,  $y = Cx$ .

### B. Observer-Based Control Design

The special structure of (14), where  $F$  is Hurwitz by construction (see Algorithm 1) and  $Hx$  can be replaced by  $y$  in the differential equation, allows us to design an observer of the plant (3) in the coordinates of the non-minimal realization, without any knowledge of the parameters in  $A$  and  $B$ . In particular, define the following dynamical system:

$$\dot{\hat{z}} = F\hat{z} + Gu + Ly, \quad (15)$$

having the same structure of (14), with user-defined matrices  $F$ ,  $G$ ,  $L$  as per Algorithm 1. Then, define the estimation error

$$\tilde{x} := x - \Pi\hat{z}. \quad (16)$$

Using (3), (12), (15), (16), the observer dynamics become

$$\begin{aligned} \dot{\hat{z}} &= F\hat{z} + Gu + L(Cx + v) \\ &= (F + LH)\hat{z} + Gu + LC\tilde{x} + Lv, \end{aligned} \quad (17)$$

and then we obtain the following estimation error dynamics:

$$\begin{aligned} \dot{\tilde{x}} &= A(\tilde{x} + \Pi\hat{z}) + Bu + Ev \\ &\quad - \Pi((F + LH)\hat{z} + Gu + LC\tilde{x} + Lv) \\ &= \tilde{\Lambda}\tilde{x} + Ev - \Pi Lv. \end{aligned} \quad (18)$$

The interconnection of (3) and (15) can be rewritten as:

$$\begin{bmatrix} \dot{\tilde{x}} \\ \dot{\hat{z}} \end{bmatrix} = \begin{bmatrix} \tilde{\Lambda} & 0_{np \times \mu} \\ LC & F + LH \end{bmatrix} \begin{bmatrix} \tilde{x} \\ \hat{z} \end{bmatrix} + \begin{bmatrix} 0_{np \times m} \\ G \end{bmatrix} u + \begin{bmatrix} E & -\Pi L \\ 0_{\mu \times q} & L \end{bmatrix} \begin{bmatrix} w \\ v \end{bmatrix}. \quad (19)$$

Since  $\tilde{\Lambda}$  in (13) is Hurwitz and the pair  $(F + LH, G)$  is controllable, the desired controller (4) is obtained by combining the observer (15) and the feedback law

$$u = K\hat{z}, \quad (20)$$

where the gain  $K \in \mathbb{R}^{m \times \mu}$  is chosen so that  $F + LH + GK$  is Hurwitz. Since  $H$  depends on the unknown plant parameters (see (12)), we cannot design  $K$  using solely the prior knowledge of  $(F + LH, G)$ . Therefore, in the following, we compute  $K$  using the input–output trajectory (6).

### C. Non-Minimal Realizations Consistent with the Dataset

Given the trajectories  $u(\cdot)$  and  $y(\cdot)$  over the interval  $[0, T]$ , we can simulate offline the observer (15), which acts as a linear filter of the data. Without loss of generality and for simplicity, we consider  $\hat{z}(0) = 0$ , so that we obtain  $\hat{z}(\cdot) : [0, T] \rightarrow \mathbb{R}^\mu$  as defined in (8). It follows that the dataset and the simulated observer trajectory  $\hat{z}(\cdot)$  satisfy (19) for almost all  $t \in [0, T]$ . Also, for all  $t \in [0, T]$ , the error  $\tilde{x}(t) = x(t) - \Pi\hat{z}(t)$  can be written explicitly using (18) and  $\tilde{x}(0) = x(0)$ :

$$\tilde{x}(t) = e^{\tilde{\Lambda}t}x(0) + \int_0^t e^{\tilde{\Lambda}(t-\tau)}(Ev(\tau) - \Pi Lv(\tau))d\tau. \quad (21)$$

We now exploit a technical result adapted from [20, Lemma 4] to rewrite  $e^{\tilde{\Lambda}t}x(0)$  in (21) in a convenient form.

**Lemma 2.** *For any  $x(0) \in \mathbb{R}^{np}$  and any  $(\Lambda, \Gamma)$  as defined in Algorithm 1, there exists  $H_0 \in \mathbb{R}^{p \times n}$  such that, for all  $t \in [0, T]$ ,  $Ce^{\tilde{\Lambda}t}x(0) = H_0\chi(t)$ , where  $\chi(t)$  is given in (8).*

Combining  $y(t) = C\tilde{x}(t) + H\hat{z}(t) + v(t)$  with (21) and Lemma 2 we obtain, for all  $t \in [0, T]$ ,

$$y(t) = \Theta^*\zeta(t) + d(t), \quad (22)$$

where  $\zeta(t)$  is defined in (8),  $\Theta^* := [H_0 \ H] \in \mathbb{R}^{p \times (n+\mu)}$  is an unknown matrix, while, for all  $t \in [0, T]$ ,

$$d(t) := v(t) + C \int_0^t e^{\tilde{\Lambda}(t-\tau)}(Ev(\tau) - \Pi Lv(\tau))d\tau. \quad (23)$$

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$$\int_0^T \begin{bmatrix} Ly(\tau) \\ -\zeta(\tau) \end{bmatrix} \begin{bmatrix} Ly(\tau) \\ -\zeta(\tau) \end{bmatrix}^\top d\tau - \left[ \begin{array}{c|c} L\Delta L^\top + FP + PF^\top + GQ + Q^\top G^\top & 0_{\mu \times n} \ P \\ \hline 0_{n \times \mu} & P \end{array} \right] \succ 0, \quad P = P^\top \succ 0. \quad (S)$$

Note that  $y(t)$  and  $\zeta(t)$  are available for measurement, while  $\Theta^*$  and  $d(t)$  are unknown. In particular, to represent the lack of knowledge of  $\Theta^*$ , we now describe the set of all matrices  $\Theta \in \mathbb{R}^{p \times (n+\mu)}$  that are consistent with the dataset (6). To obtain a set with suitable regularity properties, we impose the following assumptions on the signals appearing in (22).

**Assumption 2.** The matrix  $\Delta$  in Algorithm 1 is such that

$$\int_0^T d(\tau) d^\top(\tau) d\tau \preceq \Delta. \quad (24)$$

**Assumption 3.** The following excitation condition holds:

$$Z := \int_0^T \zeta(\tau) \zeta^\top(\tau) d\tau \succ 0. \quad (25)$$

**Remark 1.** Similar to the state-feedback scenario of [15], Assumption 2 allows us to represent the set of parameters  $\Theta$  as the solutions of a QMI and, then, exploit the results of [7] for data-driven stabilization. In Section V, we show how to compute  $\Delta$  satisfying (24) when certain prior knowledge about the noise signals  $w(\cdot)$  and  $v(\cdot)$  is available.

**Remark 2.** Assumption 3 corresponds to the full-rank conditions of the data-driven literature [4, 6, 13]. Although (25) is not necessary for introducing the non-strict LMI of [7, Cor. 4.13], it provides improved theoretical guarantees and leads to the strict LMI of [7, Thm. 4.10] which, as argued there, may be preferred in terms of numerical reliability.

For compactness of notation, define

$$\left[ \begin{array}{c|c} Y & X^\top \\ \hline X & Z \end{array} \right] := \int_0^T \left[ \begin{array}{c} y(\tau) \\ -\zeta(\tau) \end{array} \right] \left[ \begin{array}{c} y(\tau) \\ -\zeta(\tau) \end{array} \right]^\top d\tau \succeq 0, \quad (26)$$

where the lines indicate the corresponding blocks, so that  $Y \in \mathbb{R}^{p \times p}$ ,  $X \in \mathbb{R}^{(n+\mu) \times p}$ , and  $Z \in \mathbb{R}^{(n+\mu) \times (n+\mu)}$ .

Then, replace  $d(t)$  with  $y(t) - \Theta \zeta(t)$  in (24) to obtain the set of all non-minimal realizations consistent with (6):

$$\mathcal{E} := \left\{ \Theta \in \mathbb{R}^{p \times (n+\mu)} : \begin{bmatrix} I_p & \Theta \\ \Theta^\top & \end{bmatrix} N \begin{bmatrix} I_p \\ \Theta^\top \end{bmatrix} \succeq 0 \right\}, \quad (27)$$

where

$$N := \begin{bmatrix} \Delta - Y & -X^\top \\ -X & -Z \end{bmatrix}. \quad (28)$$

#### D. Data-Driven Stabilization via the Matrix S-Lemma

To solve the stabilization problem of Section II, we need to find a gain  $K$  for the feedback law (20) that stabilizes all systems contained in the uncertainty set  $\mathcal{E}$  of (27). More precisely, we want to find a pair of matrices  $P \in \mathbb{R}^{\mu \times \mu}$  and  $K \in \mathbb{R}^{m \times \mu}$  such that  $P = P^\top \succ 0$  and, for all  $\Theta \in \mathcal{E}$ ,

$$\left( F + L\Theta \begin{bmatrix} 0_{n \times \mu} \\ I_\mu \end{bmatrix} + GK \right) P + P \left( F + L\Theta \begin{bmatrix} 0_{n \times \mu} \\ I_\mu \end{bmatrix} + GK \right)^\top \prec 0, \quad (29)$$

which implies that (5) is Hurwitz by choosing the controller (15), (20). We now translate this property into an inclusion

of sets described by QMIs. Using (29), the set of all non-minimal realizations stabilized by a pair  $(P, K)$  is

$$\mathcal{C}(P, K) := \left\{ \Theta \in \mathbb{R}^{p \times (n+\mu)} : \begin{bmatrix} I_\mu \\ (L\Theta)^\top \end{bmatrix}^\top M(P, K) \begin{bmatrix} I_\mu \\ (L\Theta)^\top \end{bmatrix} \succ 0 \right\}, \quad (30)$$

where

$$M(P, K) := - \left[ \begin{array}{c|c} FP + PF^\top + GKP + PK^\top G^\top & 0_{\mu \times n} \ P \\ \hline 0_{n \times \mu} & P \\ \hline P & 0_{n+\mu} \end{array} \right]. \quad (31)$$

Then, the property that we want to impose is the following:

$$\mathcal{E} \subseteq \mathcal{C}(P, K). \quad (32)$$

The next statement, which relates (32) with the feasibility of the LMI (S) used in Algorithm 1, is the main result of this work. Its proof is based on the strict matrix S-lemma found in [7, Thm. 4.10].

**Theorem 1.** Consider Algorithm 1 and let Assumptions 1, 2, and 3 hold. Then, the following statements are equivalent:

- 1) There exist  $P \in \mathbb{R}^{\mu \times \mu}$  and  $K \in \mathbb{R}^{m \times \mu}$ , with  $P = P^\top \succ 0$ , such that  $\mathcal{E} \subseteq \mathcal{C}(P, K)$ .
- 2) There exist  $P \in \mathbb{R}^{\mu \times \mu}$  and  $Q \in \mathbb{R}^{m \times \mu}$  such that the LMI (S) is satisfied.

Moreover, if (S) is feasible, then  $K = QP^{-1}$  is such that  $F + LH + GK$  is Hurwitz and, thus, the controller (4) with gains given in (10) solves the data-driven stabilization problem of Section II.

#### IV. FEASIBILITY OF THE LMI

We now exploit Theorem 1 to provide a sufficient condition based on data such that the LMI (S) is feasible.

Under Assumption 3,  $\mathcal{E}$  can be rewritten as a bounded matrix ellipsoid. By defining  $\tilde{\Theta} := \Theta - \hat{\Theta}$ , where  $\hat{\Theta} := -X^\top Z^{-1}$  is the least-squares estimate of  $\Theta^*$ , we can change the coordinates in the QMI of (27) to obtain

$$\begin{bmatrix} I_p \\ \Theta^\top \end{bmatrix}^\top N \begin{bmatrix} I_p \\ \Theta^\top \end{bmatrix} = \begin{bmatrix} I_p \\ \tilde{\Theta}^\top \end{bmatrix}^\top \begin{bmatrix} S_N & 0_{p \times (n+\mu)} \\ 0_{(n+\mu) \times p} & -Z \end{bmatrix} \begin{bmatrix} I_p \\ \tilde{\Theta}^\top \end{bmatrix}, \quad (33)$$

where  $S_N := \Delta - Y + X^\top Z^{-1} X$  is the Schur complement of the (2, 2) block of  $N$  in (28). Thus, (27) becomes

$$\mathcal{E} = \left\{ \Theta \in \mathbb{R}^{p \times (n+\mu)} : (\Theta - \hat{\Theta}) Z (\Theta - \hat{\Theta})^\top \preceq S_N \right\}. \quad (34)$$

From (26) and  $Z \succ 0$ , it holds from the Schur complement that  $Y - X^\top Z^{-1} X \succeq 0$ . As a consequence,

$$\mathcal{E} \subseteq \hat{\mathcal{E}} := \left\{ \Theta \in \mathbb{R}^{p \times (n+\mu)} : (\Theta - \hat{\Theta}) Z (\Theta - \hat{\Theta})^\top \preceq \Delta \right\}. \quad (35)$$

Exploiting the fact that, for any  $\Theta \in \hat{\mathcal{E}}$ ,  $\lambda_{\min}(Z) (\Theta - \hat{\Theta}) (\Theta - \hat{\Theta})^\top \preceq (\Theta - \hat{\Theta}) Z (\Theta - \hat{\Theta})^\top \preceq \Delta$ , we obtain the bound

$$|\Theta - \hat{\Theta}|^2 \leq \lambda_{\max}(\Delta) / \lambda_{\min}(Z) =: \rho, \quad (36)$$

where, for  $\rho > 0$ ,  $\text{SNR} := \rho^{-1}$  is the worst-case signal-to-noise ratio according to Assumptions 2 and 3. Note that  $\Theta^* \in \mathcal{E}$ , thus it follows from the triangle inequality that

$$|\Theta - \Theta^*| = |(\Theta - \hat{\Theta}) - (\Theta^* - \hat{\Theta})| \leq 2\sqrt{\rho}. \quad (37)$$

We achieve the following feasibility condition.

**Corollary 1.** *Let Assumption 1 hold. Then, there exists  $\rho^* > 0$  such that, if the data satisfy Assumptions 2 and 3, with  $\rho := \lambda_{\max}(\Delta)/\lambda_{\min}(Z) \in [0, \rho^*]$ , then the LMI (S) is feasible.*

## V. REMARKS ON THE NOISE BOUND COMPUTATION

Note that the bound  $\Delta$  in Assumption 2 depends both on the noise signals  $w(\cdot)$ ,  $v(\cdot)$  and the parameters of the plant (3) and the observer (15). Specifically,  $d(t)$  in (23) can be split as  $d(t) = d_w(t) + d_v(t)$ , where  $d_w(t)$  and  $d_v(t)$  are the outputs of the following dynamical systems:

$$\dot{\eta}_w = \tilde{\Lambda}\eta_w + Ew, \quad d_w = C\eta_w \quad (38)$$

$$\dot{\eta}_v = \tilde{\Lambda}\eta_v - \Pi Lv, \quad d_v = C\eta_v + v, \quad (39)$$

with zero initial conditions. It is convenient to report the transfer function matrices of these systems:

$$\mathcal{G}_w(s) := \mathcal{D}_\lambda^{-1}(s)\mathcal{N}_w(s), \quad \mathcal{G}_v(s) := \mathcal{D}_\lambda^{-1}(s)\mathcal{D}(s), \quad (40)$$

where  $\mathcal{D}_\lambda$  is given in (11). Note that only  $\mathcal{G}_v(s)$  depends on unknown parameters.

Using the properties  $d(t)d(t)^\top \preceq |d(t)|^2 I_p$  and  $\|d(\cdot)\|_{2,[0,T]} \leq \|d_w(\cdot)\|_{2,[0,T]} + \|d_v(\cdot)\|_{2,[0,T]}$ , we obtain

$$\int_0^T d(\tau)d^\top(\tau)d\tau \preceq I_p(\|d_w(\cdot)\|_{2,[0,T]} + \|d_v(\cdot)\|_{2,[0,T]})^2. \quad (41)$$

Thus,  $\Delta$  in (24) can be computed using (41) if upper bounds of  $\|d_w(\cdot)\|_{2,[0,T]}$  and  $\|d_v(\cdot)\|_{2,[0,T]}$  are known. Below, we show how to compute them under certain prior knowledge of  $w(\cdot)$  and  $v(\cdot)$  and suitable properties of  $\mathcal{G}_w(s)$  and  $\mathcal{G}_v(s)$ .

### A. Bound on the Filtered Process Noise

Suppose that a scalar  $\delta_w \geq 0$  is known such that  $\|w(\cdot)\|_{2,[0,T]}^2 \leq \delta_w$ . Then, a bound on  $\|d_w(\cdot)\|_{2,[0,T]}$  is obtained by finding an upper bound  $\gamma > 0$  of the  $\mathcal{L}_2[0, T]$  gain of system (38), which ensures:

$$\|d_w(\cdot)\|_{2,[0,T]} \leq \gamma \|w(\cdot)\|_{2,[0,T]} \leq \gamma \sqrt{\delta_w}. \quad (42)$$

By [24, Thm. 3.7.4],  $\gamma$  is such that the following differential Riccati equation:

$$\dot{W} = -\tilde{\Lambda}^\top W - W\tilde{\Lambda} - \gamma^{-2}WEE^\top W - C^\top C, \quad W(T) = 0_{np}, \quad (43)$$

has a solution over  $[0, T]$ . Since all matrices in (38) are known, (43) can be used to test  $\gamma$ . Let  $\gamma_\infty > 0$  be such that  $\|\mathcal{G}_w\|_\infty < \gamma_\infty$ . Then, a conservative choice for  $\gamma$  is  $\gamma_\infty$ , because by [24, Lemma 3.7.7] (43) with  $\gamma = \gamma_\infty$  has a solution over  $[0, T]$  for all finite  $T$ . Otherwise, a tight bound can be found by searching in the interval  $[0, \gamma_\infty]$  for  $\gamma > 0$  as small as possible such that (43) has a solution over  $[0, T]$ .

### B. Bound on the Filtered Measurement Noise

In this scenario, we cannot use the same arguments as before because  $\Pi L$  in (39) depends on the unknown parameters in A. However, in the special case of single-output systems ( $p = 1$ ), we obtain the following result.

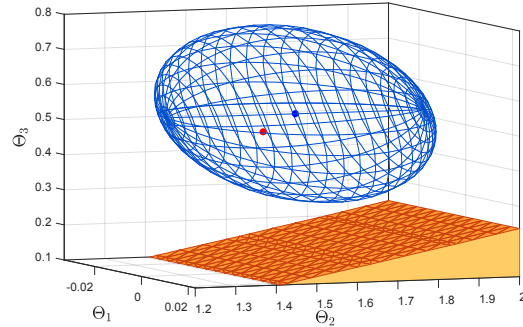
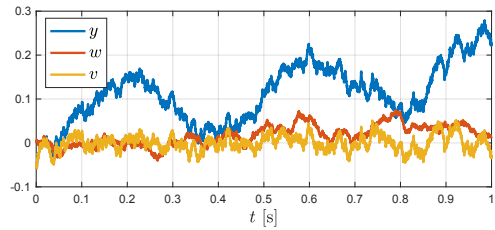


Fig. 1. Scalar example. *Top*: measured output  $y(\cdot)$  and noise signals  $w(\cdot)$  and  $v(\cdot)$ . *Bottom*: representation of  $\mathcal{E}$  (blue),  $\mathbb{R}^3 \setminus \mathcal{C}(P, K)$  (orange),  $\Theta^* = [0.15 \ 0.5 \ 0.5]$  (red dot), and  $\hat{\Theta} = [-0.0054 \ 1.6106 \ 0.5395]$  (blue dot).

**Proposition 1.** *Suppose that  $p = 1$  and let all eigenvalues of  $\Lambda$  be real and such that  $\min_{s \in \sigma(\Lambda)} |s| \geq \max_{\varphi \in \sigma(A)} |\varphi|$ . Then, it holds that  $\|\mathcal{G}_v\|_\infty = 1$ .*

Under the assumptions of Proposition 1, if a scalar  $\delta_v \geq 0$  is known such that  $\|v(\cdot)\|_{2,[0,T]}^2 \leq \delta_v$ , we obtain

$$\|d_v(\cdot)\|_{2,[0,T]} \leq \sqrt{\delta_v}. \quad (44)$$

Future work will investigate the problem of computing a bound for  $\|d_v(\cdot)\|_{2,[0,T]}$  in the multi-output scenario ( $p > 1$ ).

## VI. NUMERICAL RESULTS

We now apply Algorithm 1 to some case studies. The code has been developed in MATLAB using YALMIP [25] and MOSEK [26] and is available on Zenodo [27].

### A. Scalar System with Process and Measurement Noise

In the first example, we consider the following system:

$$\dot{x} = x + u + w, \quad y = x + v, \quad (45)$$

which allows us to graphically illustrate the set inclusion (32) used in Theorem 1. We obtain a dataset over the interval  $[0, 1]$  s by simulating (45) from  $x(0) = 0$  and applying the input  $u(\cdot) : t \mapsto \sin(5\pi t)$ . Also, we let  $\|w(\cdot)\|_{2,[0,1]}^2 = \delta_w = 0.8 \times 10^{-3}$ ,  $\|v(\cdot)\|_{2,[0,1]}^2 = \delta_v = 0.3 \times 10^{-3}$ . The noise signals and the measured output are reported in Fig. 1.

Then, we run Algorithm 1 with  $\Lambda = -2$ ,  $\Gamma = 2$  and  $\Delta = 7.1045 \times 10^{-4}$  computed according to (41), (42), (44), with  $\gamma = 0.33$  checked via (43). The resulting ellipsoid  $\mathcal{E}$  as in (27) is depicted in Fig. 1, while the computed control gain  $K$  and the matrix  $P$  obtained from the LMI (S) are

$$K = [-29.71 \ -4.87], \quad P = \begin{bmatrix} 1.26 & -5.34 \\ -5.34 & 53.25 \end{bmatrix} \times 10^{-3}, \quad (46)$$

and the eigenvalues of (5) are  $\{-2, -5.37 \pm 4.34i\}$ . In Fig. 1, we also depict the values of  $\Theta^*$  and  $\hat{\Theta}$  and the set  $\mathbb{R}^3 \setminus \mathcal{C}(P, K)$ , confirming that the property (32) holds.

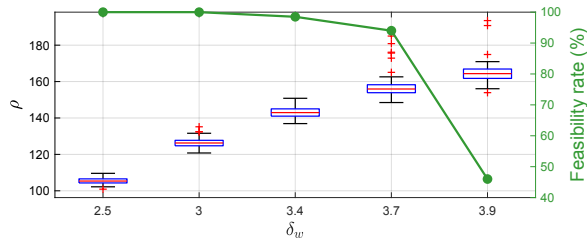


Fig. 2. Batch reactor example. Each value of  $\delta_w$  corresponds to 200 simulation runs. *Left axis*: box plot of  $\rho$  as defined in (36). *Right axis*: percentage of times the LMI ( $S$ ) is feasible.

### B. Unstable Batch Reactor with Process Noise

We now consider the model of an unstable batch reactor derived from [24, §2.6], which we write in the form (3) by letting  $m = p = n = 2$  and:

$$\begin{aligned} A_0 &= \begin{bmatrix} -20.97 & -48.63 \\ 2.643 & 5.867 \end{bmatrix}, & A_1 &= \begin{bmatrix} 5.297 & -10.47 \\ -0.2764 & 6.371 \end{bmatrix} \\ B_0 &= \begin{bmatrix} -59.44 & -12.63 \\ 12.59 & 0.8696 \end{bmatrix}, & B_1 &= \begin{bmatrix} 0 & -3.146 \\ 5.679 & 0 \end{bmatrix}. \end{aligned} \quad (47)$$

We assume that the process noise  $w \in \mathbb{R}^2$  affects the system via  $E_0 = I_2$ ,  $E_1 = 0_2$ , while we let  $v = 0$ .

In this example, we run Algorithm 1 several times to highlight the effect of the noise energy on the feasibility of the LMI ( $S$ ). Each run considers a dataset over  $[0, 3]$  s where  $x(0) = 0$ ,  $u(\cdot)$  is a sum of sinusoids (the same in each run), and  $w(\cdot)$  is randomly generated. In particular, given five values of  $\delta_w$ , we generate 200 signals uniformly extracted from the ball of radius  $\sqrt{\delta_w}$  in  $\mathcal{L}_2[0, 3]$ , approximated via a Fourier orthonormal basis comprising the constant signal and sine/cosine functions with frequencies  $2j\pi/3$ ,  $j \in \{1, \dots, 100\}$ . In all simulations, we use the filter matrices

$$\Lambda = \begin{bmatrix} 0 & -12 \\ 1 & -7 \end{bmatrix}, \quad \Gamma = \begin{bmatrix} 0 \\ 1 \end{bmatrix}. \quad (48)$$

and let  $\Delta = \gamma^2 \delta_w I_2$ , with  $\gamma = 0.07685$  checked via (43). The results are depicted in Fig. 2 and show that the feasibility of ( $S$ ) decreases as  $\rho$  in (36) increases.

## VII. CONCLUSION

We have proposed a data-driven approach to stabilize continuous-time LTI systems using the noisy input–output data collected from an experiment. The resulting algorithm involves a data-based LMI whose feasibility, under an interval excitation condition and a known noise energy bound, is both necessary and sufficient for quadratic stabilization. We have also related the LMI feasibility to the dataset’s signal-to-noise ratio and provided tuning guidelines based on the process and measurement noise finite-horizon  $\mathcal{L}_2[0, T]$  norms. Future work will extend the approach to broader classes of systems and consider datasets given by finite samples instead of a full continuous-time trajectory.

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