Derivative-Free Data-Driven Control of Continuous-Time Linear Time-Invariant Systems★

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Abstract

This paper develops a method for data-driven stabilization of continuous-time linear time-invariant systems with theoretical guarantees and no need for signal derivatives. The framework is based on linear matrix inequalities (LMIs) and illustrated in the state-feedback and single-input single-output output-feedback scenarios. Similar to discrete-time approaches, we rely solely on input and state/output measurements. In particular, we avoid differentiation by employing low-pass filters of the measured signals that, rather than approximating the derivatives, reconstruct a non-minimal realization of the plant. With access to the filter states and their derivatives, we can solve LMIs derived from sample batches of the available signals to compute a dynamic controller that stabilizes the plant. The effectiveness of the approach is showcased via numerical examples.

Keywords: Data-driven control, linear systems, linear matrix inequalities, uncertain systems

1. Introduction

Over the past decades, the paradigm of data-driven learning has gained increasing attention in the control community. This interest can be traced back to areas such as system identification (Ljung, 1999) and adaptive control (Ioannou and Sun, 2012), while the recent trends mainly focus on reinforcement learning (Sutton and Barto, 2018). A common theme across these approaches is the shift from relying on precise models to leveraging the information contained in the collected data. Recently, the dominant paradigm in data-driven control has become to compute controllers directly from data using linear matrix inequalities (LMIs) or other optimization problems, without even requiring an intermediate identification step (De Persis and Tesi, 2020). In this paper, we focus on LMI-based methods for the stabilization of continuous-time linear time-invariant systems.

Fundamental contributions to data-driven control of discrete-time systems include (De Persis and Tesi, 2020) and (Van Waarde et al., 2020a), which introduced two distinct data-based LMI formulations for state-feedback stabilization. These methodologies have also been used to address the stabilization of bilinear systems (Bisoffi et al., 2020), linear time-varying systems (Nortmann and Mylvaganam, 2020), and the linear quadratic regulator problem (De Persis and Tesi, 2020), also accounting for the effects of noise (De Persis and Tesi, 2021; Dörfler et al., 2023). Furthermore, (De Persis and Tesi, 2020) addressed also the case of output-feedback control. The integration of partial model knowledge into these approaches was explored in (Berberich et al., 2022). In this context, necessary and sufficient conditions for data informativity have been thoroughly investigated (Van Waarde et al., 2020b).

In the continuous-time scenario, the discrete-time state-feedback stabilization paradigm can be recovered via suitable sampling techniques. However, this comes at the cost of requiring samples of the state derivatives (De Persis and Tesi, 2020), causing robustness issues in the presence of noise. In (Berberich et al., 2021), LMIs inspired by (Van Waarde et al., 2020a) were derived for the design of a stabilizing gain with non-periodic sampling and noisy state-derivative estimates. Similarly, derivative estimates were employed in (Miller and Sznaier, 2022), which proposed quadratic matrix inequalities for stabilizing linear parameter-varying systems in both discrete and continuous time. Recent contributions in continuous time include the study of how sampling impacts data informativity (Eising and Cortés, 2025) and the stabilization of switched and constrained systems (Bianchi et al., 2025).

To avoid differentiation in the state-feedback case, (De Persis et al., 2023) employed integrals of the available signals, (Rapisarda et al., 2023) adopted orthogonal polynomial bases, while (Ohta and Rapisarda, 2024) proposed sampling with more general linear functionals. To the best of the authors' knowledge, no other strategy in the continuous-time literature fully removes the need for state derivatives. On the other hand, no output-feedback approach has been developed so far in this setting, where the sensitivity to noise is even more pronounced due to the need for multiple differentiations.

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This paper proposes a data-driven stabilization framework for continuous-time systems that eliminates the need for signal derivatives. The approach is first presented in the state-feedback scenario and then extended with minor modifications to the single-input single-output (SISO) output-feedback case. Instead of using derivative approximations or the methods proposed in (De Persis et al., 2023; Rapisarda et al., 2023; Ohta and Rapisarda, 2024), we define a non-minimal realization of the plant, inspired by adaptive identification (Anderson, 1974) and adaptive observer design (Narendra and Annaswamy, 1989, Ch. 4), and present it here in a state-space setting. Specifically, we process the input and state/output signals with low-pass filters that are shown to converge exponentially to an augmented system representation. Thus, rather than being used as a signal processing technique, the filters represent an observer of the non-minimal realization. Since both the state and the derivative of the filters are accessible, we employ LMIs similar to those in (De Persis and Tesi, 2020) to compute the gains of a dynamic, filter-based, stabilizing controller. Feasibility of the LMIs is ensured under suitable excitation conditions, and closed-loop stability is guaranteed regardless of the initial filter transient. Numerical examples validate the effectiveness of the approach.

The paper is organized as follows. In Section 2, we state the design problem and introduce LMI-based data-driven control. In Sections 3 and 4, we provide the algorithms for the state-feedback and the SISO scenarios. In Section 5, we show the numerical results. Finally, Section 6 concludes the paper.

Notation: We use $\mathbb{N}$, $\mathbb{R}$, and $\mathbb{C}$ to denote the sets of natural, real, and complex numbers. The identity of dimension $j \in \mathbb{N}$ is denoted with I_j. Given a symmetric matrix $M = M^\top$, $M \succ 0$ (resp. $M \succeq 0$) denotes that it is positive definite (resp. positive semidefinite). Similarly, $\prec 0$ and $\preceq 0$ are used for negative definite and semidefinite matrices.

2. Problem Statement and Preliminaries

Although the next sections will deal with both state and output feedback, for convenience, we illustrate the problem in the state-feedback scenario. Consider a linear time-invariant system of the form

$$\dot{x} = Ax + Bu, \tag{1}$$

where $x \in \mathbb{R}^n$ is the state, $u \in \mathbb{R}^m$ is the control input, and A and B are unknown matrices of appropriate dimensions. For a given initial condition $x(0)$ and some input sequence $u(t)$, suppose that the resulting input-state trajectory of (1) has been collected over an interval $[0, T]$, with $T > 0$. More specifically, suppose that the continuous-time dataset

$$(u(t), x(t)), \qquad \forall t \in [0, T], \tag{2}$$

is available. We are interested in finding an algorithm that uses (2) to compute a stabilizing controller for (1), without any prior knowledge of A and B.

We present some preliminary notions related to the existing approaches in the literature. To recover the results of data-driven stabilization of discrete-time systems, algorithms developed in a continuous-time setting involve collecting a finite batch of data of u, x, and $\dot{x}$ with a suitable sampling mechanism (De Persis and Tesi, 2020; Berberich et al., 2021; Miller and Sznaier, 2022). Given a fixed sampling time $T_\mathrm{s} := T/N$, with $N \in \mathbb{N}$, $N \geq 1$, the following batch is obtained:

$$\begin{aligned} U &:= \begin{bmatrix} u(0) & u(T_\mathrm{s}) & \cdots & u((N-1)T_\mathrm{s}) \end{bmatrix} \in \mathbb{R}^{m\times N} \\ X &:= \begin{bmatrix} x(0) & x(T_\mathrm{s}) & \cdots & x((N-1)T_\mathrm{s}) \end{bmatrix} \in \mathbb{R}^{n\times N} \\ \dot{X} &:= \begin{bmatrix} \dot{x}(0) & \dot{x}(T_\mathrm{s}) & \cdots & \dot{x}((N-1)T_\mathrm{s}) \end{bmatrix} \in \mathbb{R}^{n\times N}. \end{aligned} \tag{3}$$

We introduce a key definition used in this paper.

Definition 1. *A data batch of the form* (3) *is exciting if*

$$\operatorname{rank} \begin{bmatrix} X \\ U \end{bmatrix} = n + m. \tag{4}$$

In the scenario where (A, B) is stabilizable and the rank condition (4) holds, the data (3) can be used to construct a stabilizing feedback law for system (1) of the form $u = Kx$. In particular, it is possible to make the closed-loop system matrix $A + BK$ Hurwitz by choosing

$$K = UQ(XQ)^{-1}, \tag{5}$$

where $Q \in \mathbb{R}^{N\times n}$ is any solution of the following LMI:

$$\begin{cases} \dot{X}Q + Q^\top \dot{X}^\top \prec 0 \\ XQ = Q^\top X^\top \succ 0. \end{cases} \tag{6}$$

This result follows mutatis mutandis from the discrete-time case; see (De Persis and Tesi, 2020, Thms. 2 and 3).

We remark that, in the discrete-time scenario, only the data X and U are needed to compute K (De Persis and Tesi, 2020). Instead, the continuous-time framework requires $\dot{X}$, which cannot be reliably inferred from (2) if the data are corrupted by noise. Also, even in a noise-free scenario, approximation via finite differences leads to persistent errors in the dataset.

The direct data-driven control framework proposed in this paper avoids the need to differentiate signals both in the state-feedback setting of system (1) and in the case of output feedback for SISO systems. Note that, in the second scenario, a continuous-time algorithm corresponding to one provided in (De Persis and Tesi, 2020) would involve also higher-order derivatives of the input and the measured output.

Remark 1. *Similar to (Ohta and Rapisarda, 2024), in the following sections, we develop LMIs for the noise-free setting. On the other hand, if a bound on the noise were known, alternative LMI formulations could be employed, e.g., based on the matrix S-lemma (Van Waarde et al., 2020a). Investigating approaches to robustify the current algorithms is an important question for future research.*

3. Data-Driven Control from Input-State Data

In this section, we are interested in designing a stabilizing controller for system (1) under the following assumption.

Assumption 1. *The pair (A, B) is controllable.*

To avoid the challenge of having to measure $\dot{x}$, we propose a strategy that involves designing a filter of x and u. This filter, whose state and state derivative are accessible, is not used to approximate $\dot{x}$. Instead, it is employed to reconstruct a non-minimal realization of the plant (1), whose structure is introduced in the next subsection.

3.1. Non-Minimal System Realization

Consider the following dynamical system, having input u and output $\xi \in \mathbb{R}^n$:

$$\begin{aligned} \dot{\zeta} &= \begin{bmatrix} A & B \\ 0 & -\lambda I_m \end{bmatrix} \zeta + \begin{bmatrix} 0 \\ \gamma I_m \end{bmatrix} u \\ \xi &= \frac{1}{\gamma} \begin{bmatrix} A + \lambda I_n & B \end{bmatrix} \zeta, \end{aligned} \tag{7}$$

where $\zeta \in \mathbb{R}^{n+m}$ is the state and λ and γ, with $\gamma \neq 0$, are constant scalar tuning gains. System (7) is obtained by compactly rewriting the following system:

$$\begin{aligned} \dot{\zeta} &= -\lambda \zeta + \gamma \begin{bmatrix} \xi \\ u \end{bmatrix} \\ \xi &= \frac{1}{\gamma} \begin{bmatrix} A + \lambda I_n & B \end{bmatrix} \zeta, \end{aligned} \tag{8}$$

which, for $\lambda > 0$, acts as a low-pass filter of ξ and u. The relationship between systems (1) and (7) is provided in the next lemma, whose proof is given in the Appendix.

Lemma 1. *Under Assumption 1, for all λ and all $\gamma \neq 0$, the controllable and observable subsystem of* (7) *obeys the same dynamics of* (1), *with state $\xi \in \mathbb{R}^n$.*

In other words, Lemma 1 states that all input-state trajectories of (1) are input-output trajectories of (7), and vice versa.

It is also useful to recognize the structural property of system (7) given in Lemma 2. The proof is in the Appendix.

Lemma 2. *Under Assumption 1, for all λ and all $\gamma \neq 0$, the pair*

$$\left(\begin{bmatrix} A & B \\ 0 & -\lambda I_m \end{bmatrix}, \begin{bmatrix} 0 \\ \gamma I_m \end{bmatrix} \right) \tag{9}$$

is controllable.

3.2. Controller Design

The proposed procedure, described in Algorithm 1, is based on the following key ideas:

- Consider an input-state trajectory (10) of system (1). Choose gains λ and γ such that, in addition to $\gamma \neq 0$, $\lambda > 0$. By Lemma 1, data (10) can be seen as an input-output trajectory of system (7) with $\xi(t) = x(t)$.
- Since (7) is equivalent to (8), its behavior is simulated with (11), which is a low-pass filter of the data due to $\lambda > 0$ and can be interpreted as a state observer of (7).

Algorithm 1 Controller Design from Input-State Data

Initialization

Measured dataset:

$$(u(t), x(t)), \qquad \forall t \in [0, T]. \tag{10}$$

Tuning: $\lambda > 0$, $\gamma \neq 0$, $T_s = T/N$, with $N \in \mathbb{N}$, $N \geq 1$.

Data Batches Construction

Filter of the data: simulate for $t \in [0, T]$:

$$\dot{\hat{\zeta}}(t) = -\lambda \hat{\zeta}(t) + \gamma \begin{bmatrix} x(t) \\ u(t) \end{bmatrix}. \tag{11}$$

Initialization: $\hat{\zeta}(0) = 0$.

Sampled data batches:

$$\begin{aligned} U &:= \begin{bmatrix} u(0) & u(T_s) & \cdots & u((N-1)T_s) \end{bmatrix} \in \mathbb{R}^{m \times N} \\ Z &:= \begin{bmatrix} \hat{\zeta}(0) & \hat{\zeta}(T_s) & \cdots & \hat{\zeta}((N-1)T_s) \end{bmatrix} \in \mathbb{R}^{(n+m) \times N} \\ \dot{Z} &:= \begin{bmatrix} \dot{\hat{\zeta}}(0) & \dot{\hat{\zeta}}(T_s) & \cdots & \dot{\hat{\zeta}}((N-1)T_s) \end{bmatrix} \in \mathbb{R}^{(n+m) \times N} \\ E &:= \begin{bmatrix} x(0) & e^{-\lambda T_s} x(0) & \cdots & e^{-\lambda (N-1) T_s} x(0) \end{bmatrix} \in \mathbb{R}^{n \times N}. \end{aligned} \tag{12}$$

Stabilizing Gain Computation

LMI: find $Q \in \mathbb{R}^{N \times (n+m)}$ such that:

$$\begin{cases} \left(\dot{Z} - \left[\begin{smallmatrix} \gamma I_n \\ 0 \end{smallmatrix} \right] E \right) Q + Q^\top \left(\dot{Z} - \left[\begin{smallmatrix} \gamma I_n \\ 0 \end{smallmatrix} \right] E \right)^\top \prec 0 \\ ZQ = Q^\top Z^\top \succ 0. \end{cases} \tag{13}$$

Control gain:

$$K = UQ(ZQ)^{-1}. \tag{14}$$

Control Deployment

Control law:

$$\dot{\hat{\zeta}}_c = -\lambda \hat{\zeta}_c + \gamma \begin{bmatrix} x \\ u \end{bmatrix}, \qquad u = K \hat{\zeta}_c. \tag{15}$$

Initialization: $\hat{\zeta}_c(0) \in \mathbb{R}^{n+m}$ arbitrary.

- The simulated non-minimal state $\hat{\zeta}$ and its derivative $\dot{\hat{\zeta}}$ can be used for a data-driven control strategy that exploits realization (7) to stabilize the original plant (1). However, since A and B are unknown, $\hat{\zeta}(0)$ cannot be chosen to perfectly match the trajectories of (8) and (11). Therefore, the algorithm needs to account for the fact that (11) converges only asymptotically to a non-minimal realization of (1).

We now make the previous arguments precise. Consider the interconnection of plant (1) and filter (11), having states $(x, \hat{\zeta})$. To characterize the filter transient, define the error

$$\epsilon := x - \frac{1}{\gamma} \begin{bmatrix} A + \lambda I_n & B \end{bmatrix} \hat{\zeta}, \tag{16}$$

which originates from the fact that we cannot ensure $x(0) = \gamma^{-1} \begin{bmatrix} A + \lambda I_n & B \end{bmatrix} \hat{\zeta}(0)$. The evolution of ϵ can be computed from (1) and (11) as follows:

$$\begin{aligned}\dot{\epsilon} &= Ax + Bu - \gamma^{-1}\begin{bmatrix} A + I_n & B \end{bmatrix}\left(-\lambda\hat{\zeta} + \gamma\begin{bmatrix} x \\ u \end{bmatrix}\right) \\ &= -\lambda x + \lambda\gamma^{-1}\begin{bmatrix} A + I_n & B \end{bmatrix}\hat{\zeta} = -\lambda\epsilon.\end{aligned} \tag{17}$$

Using the change of coordinates (16), the interconnection of (1) and (11) can be represented with states $(\epsilon, \hat{\zeta})$ as:

$$\begin{aligned}\dot{\epsilon} &= -\lambda\epsilon \\ \dot{\hat{\zeta}} &= \underbrace{\begin{bmatrix} A & B \\ 0 & -\lambda I_m \end{bmatrix}}_{=:F}\hat{\zeta} + \underbrace{\begin{bmatrix} 0 \\ \gamma I_m \end{bmatrix}}_{=:G} u + \underbrace{\begin{bmatrix} \gamma I_n \\ 0 \end{bmatrix}}_{=:D}\epsilon,\end{aligned} \tag{18}$$

where the $\hat{\zeta}$-subsystem is a system with the same structure of (7) and subject to the perturbation $D\epsilon$, which converges to 0 exponentially.

From (16) and choosing $\hat{\zeta}(0) = 0$, it holds that $\epsilon(0) = x(0)$. Thus, $\epsilon(t) = e^{-\lambda t}x(0)$ can be computed for every $t \in [0, T]$. The proposed procedure involves collecting N samples of u, $\hat{\zeta}$, $\dot{\hat{\zeta}}$, and ϵ as shown in (12)[1], then solving LMI (13) and computing a control gain K from (14). The resulting controller (15) is a dynamic feedback law that incorporates the filter dynamics. Note that the state $\hat{\zeta}_c$ of (15) can be initialized arbitrarily. We are ready to present the theoretical guarantees for Algorithm 1.

Theorem 1. *Consider Algorithm 1 and let Assumption 1 hold. Then:*

1. *LMI* (13) *is feasible if the batch* (12) *is exciting, i.e.:*

$$\operatorname{rank}\begin{bmatrix} Z \\ U \end{bmatrix} = n + 2m. \tag{19}$$

2. *For any solution Q of* (13)*, the gain K computed from* (14) *is such that $F + GK$ is Hurwitz. As a consequence, the origin $(x, \hat{\zeta}_c) = 0$ of the closed-loop interconnection of plant* (1) *and controller* (15) *is globally exponentially stable.*

Proof: 1): Under Assumption 1, (F, G) is controllable by Lemma 2. Therefore, there exist matrices P, K satisfying:

$$\begin{cases}(F + GK)P + P^\top(F + GK)^\top \prec 0 \\ P = P^\top \succ 0.\end{cases} \tag{20}$$

Given any P, K satisfying (20), condition (19) implies that there exists a matrix M_K such that (De Persis and Tesi, 2020, Thm. 2):

$$\begin{bmatrix} I_{n+m} \\ K \end{bmatrix} = \begin{bmatrix} Z \\ U \end{bmatrix} M_K. \tag{21}$$

Notice that $\dot{Z} = FZ + GU + DE$ from (18). Then, using (21), it holds that:

$$\begin{aligned}F + GK &= \begin{bmatrix} F & G \end{bmatrix}\begin{bmatrix} I_{n+m} \\ K \end{bmatrix} = \begin{bmatrix} F & G \end{bmatrix}\begin{bmatrix} Z \\ U \end{bmatrix} M_K \\ &= (\dot{Z} - DE)M_K.\end{aligned} \tag{22}$$

[1] By sampling $\dot{\hat{\zeta}}$, we intend that we measure the right-hand side of (11). The same approach is used for the sampled derivatives of the next section.

Combining (20) and (22), we obtain:

$$\begin{cases}(\dot{Z} - DE)M_K P + P^\top M_K^\top(\dot{Z} - DE)^\top \prec 0 \\ P = P^\top \succ 0.\end{cases} \tag{23}$$

Let $Q = M_K P$ and notice that (21) implies that $P = ZM_K P = ZQ$. Replacing these identities in (23), we obtain (13).

2): Suppose that there exists Q that satisfies (13). Since ZQ is symmetric and positive definite, Z has full row rank. Also, $Z^\dagger := Q(ZQ)^{-1}$ is a right inverse of Z. Using $\dot{Z} = FZ + GU + DE$ and the above properties, we have that

$$F = (\dot{Z} - DE - GU)Z^\dagger. \tag{24}$$

Using (14) and (24), we obtain:

$$F + GK = (\dot{Z} - DE - GU)Z^\dagger + GUZ^\dagger = (\dot{Z} - DE)Z^\dagger. \tag{25}$$

Let $P = ZQ$. Then, the first inequality of (13) reads as:

$$(\dot{Z} - DE)Q(ZQ)^{-1}P + P(ZQ)^{-1}Q^\top(\dot{Z} - DE)^\top \prec 0, \tag{26}$$

implying that $(\dot{Z} - DE)Q(ZQ)^{-1} = (\dot{Z} - DE)Z^\dagger = F + GK$ is Hurwitz.

To conclude the proof, we compactly rewrite the closed-loop interconnection of (1) and (15) using (18) and $u = K\hat{\zeta}_c$:

$$\begin{aligned}\dot{\epsilon} &= -\lambda\epsilon \\ \dot{\hat{\zeta}}_c &= (F + GK)\hat{\zeta}_c + D\epsilon.\end{aligned} \tag{27}$$

Since $F + GK$ is Hurwitz and $\lambda > 0$, global exponential stability of (27) follows from standard results for cascaded linear systems. □

Remark 2. *Due to space limitations, we do not formally study how to ensure* (19)*. However, we provide some insights:*

- *System (7) is controllable by Lemma 2, so $(\zeta(t), u(t))$ is persistently exciting by (Nordström and Sastry, 1987, Thms. 1 and 2) if the input $u(t)$ is sufficiently rich. Since $\hat{\zeta}(t) - \zeta(t) \to 0$ exponentially, for a dataset length $T > 0$ sufficiently large, there exists $\mu > 0$ such that:*

$$\int_0^T \begin{bmatrix} \hat{\zeta}(\tau) \\ u(\tau) \end{bmatrix}\begin{bmatrix} \hat{\zeta}(\tau) \\ u(\tau) \end{bmatrix}^\top \mathrm{d}\tau \geq \mu I_{n+2m}. \tag{28}$$

- *Under sufficient smoothness of the involved signals and sufficiently small sampling time $T_s > 0$ (see, e.g., (Eising and Cortés, 2025, Lemma IV.3)),* (28) *implies* (19)*.*

4. Data-Driven Control from Input-Output Data

To highlight the parallelism with the state-feedback scenario, we slightly abuse the notation of Section 3 by adopting here similar symbols. Consider a single-input single-output system of the form

$$\begin{aligned}\dot{x} &= Ax + bu \\ y &= c^\top x,\end{aligned} \tag{29}$$

where $x \in \mathbb{R}^n$ is the unmeasured state, of which we only know the dimension n, $u \in \mathbb{R}$ is the control input, $y \in \mathbb{R}$ is the measured output, and $A \in \mathbb{R}^{n\times n}$, $b \in \mathbb{R}^n$, and $c \in \mathbb{R}^n$ are matrices with unknown entries that satisfy the following assumption.

Assumption 2. *The pair (A, b) is controllable and the pair $(c^\top, A)$ is observable.*

Compared to Section 3, the additional challenge of this scenario is that only the output y is available instead of the state x. However, once again, it is possible to introduce a filter of the data that reconstructs a non-minimal realization of the plant, thus enabling the application of data-driven control without measuring derivatives.

4.1. Non-Minimal System Realization

Define the following dynamical system, having input u and output $\eta \in \mathbb{R}$:

$$\begin{aligned} \dot{\zeta} &= \begin{bmatrix} \Lambda + \ell\theta_1^\top & \ell\theta_2^\top \\ 0 & \Lambda \end{bmatrix} \zeta + \begin{bmatrix} 0 \\ \ell \end{bmatrix} u \\ \eta &= \begin{bmatrix} \theta_1^\top & \theta_2^\top \end{bmatrix} \zeta, \end{aligned} \tag{30}$$

where $\zeta \in \mathbb{R}^{2n}$ is the state , $\Lambda \in \mathbb{R}^{n\times n}$ and $\ell \in \mathbb{R}^n$ are constant tuning gains, and θ_1, $\theta_2 \in \mathbb{R}^n$ are constant vectors whose values, unknown for design, will be derived from A, b, c, Λ, and ℓ to match the input-output behavior of systems (29) and (30). System (30) is obtained from the following representation, also used in the literature of adaptive observers (Narendra and Annaswamy, 1989, Ch. 4):

$$\begin{aligned} \dot{\zeta} &= \begin{bmatrix} \Lambda & 0 \\ 0 & \Lambda \end{bmatrix} \zeta + \begin{bmatrix} \ell & 0 \\ 0 & \ell \end{bmatrix} \begin{bmatrix} \eta \\ u \end{bmatrix} \\ \eta &= \begin{bmatrix} \theta_1^\top & \theta_2^\top \end{bmatrix} \zeta \end{aligned} \tag{31}$$

which can be interpreted as a filter of η and u.

In order to extend the properties found in Section 3.1 to the input-output scenario, vectors θ_1 and θ_2 can be chosen according to the following fundamental result, whose proof is given in the Appendix.

Lemma 3. *Under Assumption 2, for all controllable pairs (Λ, ℓ) such that Λ has distinct eigenvalues, there exist a full rank matrix $\Pi \in \mathbb{R}^{n\times 2n}$ and vectors θ_1, $\theta_2 \in \mathbb{R}^n$ such that:*

$$\begin{gathered} \Pi \begin{bmatrix} \Lambda + \ell\theta_1^\top & \ell\theta_2^\top \\ 0 & \Lambda \end{bmatrix} = A\Pi, \quad \Pi \begin{bmatrix} 0 \\ \ell \end{bmatrix} = b \\ \begin{bmatrix} \theta_1^\top & \theta_2^\top \end{bmatrix} = c^\top \Pi. \end{gathered} \tag{32}$$

In particular, $A - \Pi_1 \ell c^\top$ and Λ are similar, where Π_1 is the matrix given by the first n columns of Π.

The next results are equivalent to Lemmas 1 and 2. The proofs are given in the Appendix.

Lemma 4. *Given the assumptions and matrices Π, θ_1, θ_2 of Lemma 3, the controllable and observable subsystem of (30) obeys the same dynamics of (29), with state $\Pi\zeta \in \mathbb{R}^n$ and output $\eta \in \mathbb{R}$.*

Lemma 5. *Given the assumptions and matrices Π, θ_1, θ_2 of Lemma 3, the pair*

$$\left(\begin{bmatrix} \Lambda + \ell\theta_1^\top & \ell\theta_2^\top \\ 0 & \Lambda \end{bmatrix}, \begin{bmatrix} 0 \\ \ell \end{bmatrix} \right) \tag{33}$$

is controllable.

4.2. Controller Design

The procedure presented in Algorithm 2 follows similar ideas to those illustrated in the state-feedback scenario.

Given an input-output trajectory (34) of (29), define system (35), replicating dynamics (31) with $\eta(t) = y(t)$. In (35), we choose Λ diagonal with distinct negative diagonal entries and ℓ with all non-zero entries. As a consequence, Λ is Hurwitz and thus (35) is a low-pass filter of the input-output data (34). Also, Λ has distinct eigenvalues and it can be verified with the

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Algorithm 2 Controller Design from Input-Output Data

Initialization

Measured dataset:

$$(u(t), y(t)), \qquad \forall t \in [0, T]. \tag{34}$$

Tuning: $\Lambda = \operatorname{diag}(-\lambda_1, \dots, -\lambda_n)$, with $0 < \lambda_1 < \dots < \lambda_n$, $\ell = \operatorname{col}(\gamma_1, \dots, \gamma_n)$, with $\gamma_1 \neq 0, \dots, \gamma_n \neq 0$, $T_s = T/N$, with $N \in \mathbb{N}$, $N \geq 1$.

Data Batches Construction

Filter of the data: simulate for $t \in [0, T]$:

$$\dot{\hat{\zeta}}(t) = \begin{bmatrix} \Lambda & 0 \\ 0 & \Lambda \end{bmatrix} \hat{\zeta}(t) + \begin{bmatrix} \ell & 0 \\ 0 & \ell \end{bmatrix} \begin{bmatrix} y(t) \\ u(t) \end{bmatrix}. \tag{35}$$

Initialization: $\hat{\zeta}(0) = 0$.

Auxiliary dynamics: simulate for $t \in [0, T]$:

$$\dot{\chi}(t) = \Lambda \chi(t). \tag{36}$$

Initialization: $\chi(0) = [1 \ \cdots \ 1]^\top$.

Sampled data batches:

$$\begin{aligned} U &:= \begin{bmatrix} u(0) & u(T_s) & \cdots & u((N-1)T_s) \end{bmatrix} \in \mathbb{R}^{1\times N} \\ Z_a &:= \begin{bmatrix} \chi(0) & \chi(T_s) & \cdots & \chi((N-1)T_s) \\ \hat{\zeta}(0) & \hat{\zeta}(T_s) & \cdots & \hat{\zeta}((N-1)T_s) \end{bmatrix} \in \mathbb{R}^{3n\times N} \\ \dot{Z}_a &:= \begin{bmatrix} \dot{\chi}(0) & \dot{\chi}(T_s) & \cdots & \dot{\chi}((N-1)T_s) \\ \dot{\hat{\zeta}}(0) & \dot{\hat{\zeta}}(T_s) & \cdots & \dot{\hat{\zeta}}((N-1)T_s) \end{bmatrix} \in \mathbb{R}^{3n\times N}. \end{aligned} \tag{37}$$

Stabilizing Gain Computation

LMI: find $Q \in \mathbb{R}^{N\times 3n}$ such that:

$$\begin{cases} \dot{Z}_a Q + Q^\top \dot{Z}_a^\top \prec 0 \\ Z_a Q = Q^\top Z_a^\top \succ 0. \end{cases} \tag{38}$$

Control gain:

$$K = UQ(Z_a Q)^{-1} \begin{bmatrix} 0 \\ I_{2n} \end{bmatrix}. \tag{39}$$

Control Deployment

Control law:

$$\dot{\hat{\zeta}}_c = \begin{bmatrix} \Lambda & 0 \\ 0 & \Lambda \end{bmatrix} \hat{\zeta}_c + \begin{bmatrix} \ell & 0 \\ 0 & \ell \end{bmatrix} \begin{bmatrix} y \\ u \end{bmatrix}, \quad u = K\hat{\zeta}_c. \tag{40}$$

Initialization: $\hat{\zeta}_c(0) \in \mathbb{R}^{2n}$ arbitrary.

PBH test that pair (Λ, ℓ) is controllable. Thus, Lemmas 3, 4, and 5 hold. The proposed design can be derived for more general choices of Λ and ℓ, although at the expense of increased notational burden.

Consider Π, θ_1, θ_2 from Lemma 3, then define:

$$\epsilon \coloneqq x - \Pi\hat{\zeta}, \tag{41}$$

and note that $\epsilon(0) = x(0)$ since we choose $\hat{\zeta}(0) = 0$. The dynamics of ϵ are computed from (29), (32), (35), and (41) as follows:

$$\begin{aligned}\dot{\epsilon} &= Ax + bu - \Pi\begin{bmatrix}\Lambda & 0\\ 0 & \Lambda\end{bmatrix}\hat{\zeta} - \Pi\begin{bmatrix}\ell & 0\\ 0 & \ell\end{bmatrix}\begin{bmatrix}c^\top x\\ u\end{bmatrix}\\ &= (A - \Pi_1\ell c^\top)\epsilon + \left(A\Pi - \Pi\begin{bmatrix}\Lambda + \ell\theta_1^\top & \ell\theta_2^\top\\ 0 & \Lambda\end{bmatrix}\right)\hat{\zeta}\\ &= H\Lambda H^{-1}\epsilon,\end{aligned} \tag{42}$$

where H is a non-singular matrix that exists due to Λ and $A - \Pi_1\ell c^\top$ being similar by Lemma 3. We can write the interconnection of plant (29) and filters (35) using the change of coordinates (41) and (32), leading to

$$\begin{aligned}\dot{\epsilon} &= H\Lambda H^{-1}\epsilon\\ \dot{\hat{\zeta}} &= \underbrace{\begin{bmatrix}\Lambda + \ell\theta_1^\top & \ell\theta_2^\top\\ 0 & \Lambda\end{bmatrix}}_{=:F}\hat{\zeta} + \underbrace{\begin{bmatrix}0\\ \ell\end{bmatrix}}_{=:g}u + \underbrace{\begin{bmatrix}\ell c^\top\\ 0\end{bmatrix}}_{=:D}\epsilon,\end{aligned} \tag{43}$$

which has the same structure of (18).

Contrary to Section 3, $D\epsilon$ is not available in the output-feedback scenario. Since $\epsilon \to 0$ exponentially, a simple approach would be to sample u, $\hat{\zeta}$, and $\dot{\hat{\zeta}}$ after a sufficiently long time to make the perturbation $D\epsilon$ small enough. This method, however, would cause an inefficient use of data and would not be rigorous as ϵ can be arbitrarily large due to A, b, c, and $x(0)$ being unknown. In the following, instead, we propose an approach that compensates $D\epsilon$ exactly without any need for a waiting time.

From (43) and $\epsilon(0) = x(0)$, $\epsilon(t)$ can be computed as

$$\epsilon(t) = e^{H\Lambda H^{-1}t}\epsilon(0) = He^{\Lambda t}H^{-1}x(0) = L\chi(t), \tag{44}$$

where $L \in \mathbb{R}^{n\times n}$ is an unknown matrix depending on H and $x(0)$[2], while $\chi(t) \coloneqq [e^{-\lambda_1 t} \ \cdots \ e^{-\lambda_n t}]^\top \in \mathbb{R}^n$. Note that $\chi(t)$ obeys dynamics (36), with $\chi(0) = [1 \ \cdots \ 1]^\top$. Thus, the sequence $(u(t), \hat{\zeta}(t))$ obtained from (34), (35) satisfies for all $t \in [0, T]$ the following differential equation:

$$\begin{bmatrix}\dot{\chi}\\ \dot{\hat{\zeta}}\end{bmatrix} = \begin{bmatrix}\Lambda & 0\\ DL & F\end{bmatrix}\begin{bmatrix}\chi\\ \hat{\zeta}\end{bmatrix} + \begin{bmatrix}0\\ g\end{bmatrix}u, \tag{45}$$

with initial conditions $\chi(0) = [1 \ \cdots \ 1]^\top$, $\hat{\zeta}(0) = 0$.

Note that the state and state derivative of (45) are available. We can then sample u, $(\chi, \hat{\zeta})$, and $(\dot{\chi}, \dot{\hat{\zeta}})$ to obtain the batches (37), which are used to design a feedback law for the plant (29). In particular, solving the LMI in (38) and computing the control gain K via (39) yields the observer-based controller summarized in (40). The next result provides the theoretical guarantees for Algorithm 2.

Theorem 2. *Consider Algorithm 2 and let Assumption 2 hold. Then:*

1. *LMI* (38) *is feasible if the batch* (37) *is exciting, i.e.:*
$$\operatorname{rank}\begin{bmatrix}Z_\mathrm{a}\\ U\end{bmatrix} = 3n + 1. \tag{46}$$
2. *For any solution Q of* (38)*, the gain K computed from* (39) *is such that $F + gK$ is Hurwitz. As a consequence, the origin $(x, \hat{\zeta}_\mathrm{c}) = 0$ of the closed-loop interconnection of plant* (29) *and controller* (40) *is globally exponentially stable.*

Proof: Since we follow similar arguments to those in Theorem 1, we only provide a sketch.

1): Consider system (45). Since (F, g) is controllable by Lemma 5 and Λ is Hurwitz, the pair

$$F_\mathrm{a} \coloneqq \begin{bmatrix}\Lambda & 0\\ DL & F\end{bmatrix}, \quad g_\mathrm{a} \coloneqq \begin{bmatrix}0\\ g\end{bmatrix} \tag{47}$$

is stabilizable. As a consequence, we ensure the existence of a matrix Q satisfying (38) by following the same steps of the proof of Theorem 1, part 1, where we replace Z, F, G with Z_a, F_a, g_a and exploit the new rank condition (46) and relationship $\dot{Z}_\mathrm{a} = F_\mathrm{a}Z_\mathrm{a} + g_\mathrm{a}U$ in place of (19) and $\dot{Z} = FZ + GU + DE$.

2): Following the same steps of the proof of Theorem 1, part 2, with the same modifications as before, we exploit (38) to ensure that $K_\mathrm{a} \coloneqq [K_\chi \ K] = UQ(Z_\mathrm{a}Q)^{-1}$ is such that

$$F_\mathrm{a} + g_\mathrm{a}K_\mathrm{a} = \begin{bmatrix}\Lambda & 0\\ DL + gK_\chi & F + gK\end{bmatrix} \tag{48}$$

is Hurwitz. We conclude the proof by noticing from (48) that K in (39) ensures that $F + gK$ is Hurwitz, thus we can follow the arguments of Theorem 1, now applied to (43), to prove global exponential stability of the interconnection of (29) and (40). □

Remark 3. *$V \coloneqq [\chi(0) \ \cdots \ \chi((N-1)T_\mathrm{s})]$ is a Vandermonde matrix with roots $e^{-\lambda_1 T_\mathrm{s}}, \ldots, e^{-\lambda_n T_\mathrm{s}}$, so it has full row rank when $N \geq n$. Define $Z \coloneqq [\hat{\zeta}(0) \ \cdots \ \hat{\zeta}((N-1)T_\mathrm{s})]$. Then,* (46) *holds if $[Z^\top \ U^\top]^\top$ has full row rank and is linearly independent from V. For the full rank of $[Z^\top \ U^\top]^\top$, we refer to Remark 2. To give an intuition on the second requirement, let $\hat{\zeta}(t)$ and $u(t)$ be the sum of p sinusoids at distinct frequencies. From* $\sin(\omega t) = (e^{\mathrm{i}\omega t} - e^{-\mathrm{i}\omega t})/(2\mathrm{i})$, $\cos(\omega t) = (e^{\mathrm{i}\omega t} + e^{-\mathrm{i}\omega t})/2$*:*

$$\begin{bmatrix}Z\\ U\end{bmatrix} = \Psi W = \Psi\begin{bmatrix}1 & e^{\mathrm{i}\omega_1 T_\mathrm{s}} & \ldots & e^{\mathrm{i}\omega_1(N-1)T_\mathrm{s}}\\ 1 & e^{-\mathrm{i}\omega_1 T_\mathrm{s}} & \ldots & e^{-\mathrm{i}\omega_1(N-1)T_\mathrm{s}}\\ \vdots & \vdots & & \vdots\\ 1 & e^{\mathrm{i}\omega_p T_\mathrm{s}} & \ldots & e^{\mathrm{i}\omega_p(N-1)T_\mathrm{s}}\\ 1 & e^{-\mathrm{i}\omega_p T_\mathrm{s}} & \ldots & e^{-\mathrm{i}\omega_p(N-1)T_\mathrm{s}}\end{bmatrix}, \tag{49}$$

where $\Psi \in \mathbb{C}^{(2n+1)\times 2p}$ and W is a Vandermonde matrix. As a consequence, $N \geq n + 2p$ ensures that W and V are linearly independent, implying that each non-zero row of ΨW is linearly independent from V.

[2] $L \coloneqq ((H^{-1}x(0))^\top \otimes H)\,\mathrm{diag}(\mathrm{e}_1, \ldots, \mathrm{e}_n)$, where $\otimes$ denotes the Kronecker product and $(\mathrm{e}_1, \ldots, \mathrm{e}_n)$ is the orthonormal basis of $\mathbb{R}^n$.

5. Numerical Examples

Algorithms 1 and 2 have been implemented in MATLAB using YALMIP (Löfberg, 2004) and MOSEK (ApS, 2024) to solve the LMIs. The developed code is available on Zenodo (Bosso and Borghesi, 2025).

It is worth mentioning that, in Algorithm 1, the LMI (13) has been implemented in the following equivalent form, which reformulates the constraints $ZQ = Q^\top Z^\top \succ 0$:

$$\begin{cases} \left(\dot{Z} - \left[\begin{smallmatrix}\gamma I_n \\ 0\end{smallmatrix}\right] E\right) Q + Q^\top \left(\dot{Z} - \left[\begin{smallmatrix}\gamma I_n \\ 0\end{smallmatrix}\right] E\right)^\top \prec 0 \\ P = P^\top \succ 0 \\ ZQ = P, \end{cases} \tag{50}$$

with decision variables $Q \in \mathbb{R}^{N\times(n+m)}$ and $P \in \mathbb{R}^{(n+m)\times(n+m)}$. Similarly, in Algorithm 2, LMI (38) has been implemented as

$$\begin{cases} \dot{Z}_{\mathrm{a}} Q + Q^\top \dot{Z}_{\mathrm{a}}^\top \prec 0 \\ P = P^\top \succ 0 \\ Z_{\mathrm{a}} Q = P, \end{cases} \tag{51}$$

with decision variables $Q \in \mathbb{R}^{N\times 3n}$ and $P \in \mathbb{R}^{3n\times 3n}$.

5.1. Design with Input-State Data

We consider the continuous-time linearized model of an unstable batch reactor given in (Walsh and Ye, 2001), also used in (De Persis and Tesi, 2020) after time discretization. The system matrices are

$$A = \begin{bmatrix} 1.38 & -0.2077 & 6.715 & -5.676 \\ -0.5814 & -4.29 & 0 & 0.675 \\ 1.067 & 4.273 & -6.654 & 5.893 \\ 0.048 & 4.273 & 1.343 & -2.104 \end{bmatrix}$$
$$B = \begin{bmatrix} 0 & 0 \\ 5.679 & 0 \\ 1.136 & -3.146 \\ 1.136 & 0 \end{bmatrix}, \tag{52}$$

where (A, B) is controllable and the eigenvalues of A are $\{-8.67, -5.06, 0.0635, 1.99\}$. We consider an exploration interval of length $T = 1.5$ s, where we apply a sum of 4 sinusoids to both entries of u and select 8 distinct frequencies to ensure informative data. We choose filter gains $\lambda = \gamma = 1$ and collect the data with sampling time $T_{\mathrm{s}} = 0.1$ s.

Algorithm 1 has been extensively tested for random initial conditions $x(0)$, with each entry extracted from the uniform distribution $\mathcal{U}(-1, 1)$, returning each time a stabilizing controller. For $x(0) = [0.1233 \;\; -0.7076 \;\; 0.4464 \;\; -0.8085]^\top$, we obtain the gain

$$K = \begin{bmatrix} 10.71 & -16.82 & 7.792 & -7.581 & 2.823 & -7.857 \\ 17.45 & -4.386 & 45.95 & -33.59 & 8.814 & -3.416 \end{bmatrix}, \tag{53}$$

which places the eigenvalues of the closed-loop matrix $F + GK$ in $\{-2.831, -2.725 \pm 7.749\mathrm{i}, -2.63 \pm 14.28\mathrm{i}, -0.72\}$.

5.2. Design with Input-Output Data

We consider a non-minimum phase SISO system having input-output behavior specified by the transfer function

$$c^\top (sI - A)^{-1} b = \frac{s-1}{s(s^2+4)}, \tag{54}$$

for which we choose a minimal realization in controller canonical form. We perform an exploration of $T = 2$ s with an input u given by the sum of 4 sinusoids at distinct frequencies. We choose filter parameters $\Lambda = \mathrm{diag}(-1, -2, -3)$, $\ell = \mathrm{col}(1, 2, 3)$ and sampling time $T_{\mathrm{s}} = 0.1$ s.

Similar to the previous case, Algorithm 2 has been extensively tested with random initial conditions so that each entry of $x(0)$ is extracted from the uniform distribution $\mathcal{U}(-5, 5)$. In each test, the procedure returned a stabilizing controller. For $x(0) = [-2.1002 \;\; 4.5808 \;\; 2.2305]^\top$, we obtain the gain

$$K = \begin{bmatrix} -0.3 & 2.882 & -2.23 & -0.095 & -0.56 & 1.081 \end{bmatrix}, \tag{55}$$

which places the eigenvalues of $F + gK$ in $\{-1.978, -0.73 \pm 0.7\mathrm{i}, -0.237, -0.148 \pm 2.095\mathrm{i}\}$.

6. Conclusion

We addressed the problem of data-driven stabilization of unknown continuous-time linear time-invariant systems by proposing a framework that combines signal filtering with LMIs. Specifically, we employed low-pass filters that reconstruct a non-minimal realization of the plant. Then, LMIs inspired by those of (De Persis and Tesi, 2020) and based on the non-minimal realization have been used to compute the gains of a dynamic, filter-based controller. This approach circumvents the need for signal derivatives without resorting to numerical techniques like finite differences. We remark that the proposed LMIs have been developed for the noise-free setting. Therefore, future work will address the case of noisy data, exploiting techniques such as (Van Waarde et al., 2020a). Other research directions include extending the method to multi-input multi-output and nonlinear systems, as well as exploring the conditions to ensure exciting data.

7. Appendix

7.1. Proof of Lemma 1

We prove the result by constructing the Kalman observability decomposition of (7).

Define $\Pi := \gamma^{-1}[A + \lambda I_n \;\; B]$, so that $\xi = \Pi\zeta$ in (7). Since (A, B) is controllable by Assumption 1, rank $\Pi = n$ from a direct application of the PBH test. Additionally, note that

$$\dot{\xi} = \Pi \begin{bmatrix} A & B \\ 0 & -\lambda I_m \end{bmatrix} \zeta + \Pi \begin{bmatrix} 0 \\ \gamma I_m \end{bmatrix} u = A\xi + Bu. \tag{56}$$

Let $\Xi \in \mathbb{R}^{m\times(n+m)}$ be composed of m row vectors linearly independent of those of Π. Then, define the change of coordinates

$\zeta \mapsto \text{col}(\xi, \xi_{\bar{o}})$, where $\xi_{\bar{o}} := \Xi\zeta$. In the new coordinates, the plant reads as:

$$\begin{aligned} \begin{bmatrix} \dot{\xi} \\ \dot{\xi}_{\bar{o}} \end{bmatrix} &= \begin{bmatrix} A & 0 \\ A_\Delta & A_{\bar{o}} \end{bmatrix} \begin{bmatrix} \xi \\ \xi_{\bar{o}} \end{bmatrix} + \begin{bmatrix} B \\ B_{\bar{o}} \end{bmatrix} u \\ \xi &= \begin{bmatrix} I_n & 0 \end{bmatrix} \begin{bmatrix} \xi \\ \xi_{\bar{o}} \end{bmatrix}, \end{aligned} \tag{57}$$

for some matrices $A_{\bar{o}}$, A_Δ, $B_{\bar{o}}$. Therefore, the observable states ξ of (57) obey (1), which is controllable by Assumption 1. □

7.2. *Proof of Lemma 2*

We use the PBH test, which requires that

$$\text{rank} \begin{bmatrix} sI_n - A & -B & 0 \\ 0 & (s+\lambda)I_m & \gamma I_m \end{bmatrix} = n + m \tag{58}$$

for all $s \in \sigma(A) \cup \{-\lambda\}$. From Assumption 1, the first n rows are linearly independent for all $s \in \sigma(A) \cup \{-\lambda\}$. Therefore, the result follows by noticing that the remaining m rows are linearly independent from the previous ones. □

7.3. *Proof of Lemma 3*

Define $\Pi := [\Pi_1\ \Pi_2]$, with $\Pi_1, \Pi_2 \in \mathbb{R}^{n\times n}$. Then, (32) can be written as

$$\begin{aligned} \Pi_1 \Lambda &= (A - \Pi_1 \ell c^\top)\Pi_1 \\ \theta_1^\top &= c^\top \Pi_1 \end{aligned} \tag{59}$$

and

$$\begin{aligned} \Pi_2 \Lambda &= (A - \Pi_1 \ell c^\top)\Pi_2 \\ \Pi_2 \ell &= b, \quad \theta_2^\top = c^\top \Pi_2, \end{aligned} \tag{60}$$

where we notice that $A - \Pi_1 \ell c^\top$ appears in both equations. Since pair $(c^\top, A)$ is observable, we can define ψ as the unique vector such that the spectra of $A - \psi c^\top$ and Λ coincide. Since all eigenvalues of Λ are distinct, $A - \psi c^\top$ and Λ are similar, so there exists an invertible matrix H such that $H\Lambda H^{-1} = A - \psi c^\top$. Define the following equations:

$$\begin{aligned} \Pi_1 \Lambda &= (A - \psi c^\top)\Pi_1 \\ \Pi_1 \ell &= \psi, \quad \theta_1^\top = c^\top \Pi_1, \end{aligned} \tag{61}$$

$$\begin{aligned} \Pi_2 \Lambda &= (A - \psi c^\top)\Pi_2 \\ \Pi_2 \ell &= b, \quad \theta_2^\top = c^\top \Pi_2. \end{aligned} \tag{62}$$

Any solution of (61), (62) is also a solution of (59), (60). Also, θ_1 and θ_2 can be computed after Π_1 and Π_2 have been found. By introducing the similarity transformation H in (61), (62), we obtain the following two equations:

$$X_i \Lambda = \Lambda X_i, \quad X_i \ell = h_i, \qquad i \in \{1, 2\} \tag{63}$$

where $h_1 = H^{-1}\psi$, $h_2 = H^{-1}b$, and $\Pi_i = HX_i$. To solve (63), we use the same arguments of (Serrani et al., 2000, Prop. 4.1). Since Λ has distinct eigenvalues, by (Gantmakher, 1960, Pag. 222), any matrix X_i that commutes with Λ, i.e., that satisfies equation $X_i\Lambda = \Lambda X_i$ of (63), can be expressed as

$$X_i = \sum_{j=0}^{n-1} \mu_{ij} \Lambda^j, \tag{64}$$

with free parameters $(\mu_{i0}, \ldots, \mu_{i(n-1)})$. By replacing (64) in the second equation of (63), it holds that

$$\sum_{j=0}^{n-1} \mu_{ij} \Lambda^j \ell = R\mu_i = h_i, \tag{65}$$

where $R := [\ell\ \Lambda\ell\ \cdots\ \Lambda^{n-1}\ell]$ and $\mu_i := \text{col}(\mu_{i0}, \ldots, \mu_{i(n-1)})$. Since (Λ, ℓ) is controllable, R is invertible, therefore $\mu_i = R^{-1}h_i$ and, thus, X_i is uniquely defined. The previous steps prove that a solution Π, θ_1, θ_2 exists for equations (32) and that $A - \Pi_1 \ell c^\top$ and Λ are similar.

To show that Π has full row rank, pre-multiply by A the second equation of (32):

$$A\Pi \begin{bmatrix} 0 \\ \ell \end{bmatrix} = \Pi \begin{bmatrix} \Lambda + \ell\theta_1^\top & \ell\theta_2^\top \\ 0 & \Lambda \end{bmatrix} \begin{bmatrix} 0 \\ \ell \end{bmatrix} = Ab. \tag{66}$$

Repeat this process and stack the resulting vectors to obtain:

$$\Pi M = \begin{bmatrix} b & Ab & \cdots & A^{n-1}b \end{bmatrix}, \tag{67}$$

where

$$M := \begin{bmatrix} \begin{bmatrix} 0 \\ \ell \end{bmatrix} & \cdots & \begin{bmatrix} \Lambda + \ell\theta_1^\top & \ell\theta_2^\top \\ 0 & \Lambda \end{bmatrix}^{n-1} \begin{bmatrix} 0 \\ \ell \end{bmatrix} \end{bmatrix}. \tag{68}$$

Since (A, b) is controllable, from (67), $\text{rank}(\Pi M) = n$. From $n = \text{rank}(\Pi M) \le \min\{\text{rank}(\Pi), \text{rank}(M)\} \le n$, we conclude that $\text{rank}(\Pi) = n$. □

7.4. *Proof of Lemma 4*

Given Π, θ_1, θ_2 from Lemma 3, define $\xi := \Pi\zeta$. Then:

$$\begin{aligned} \dot{\xi} &= \Pi\left(\begin{bmatrix} \Lambda + \ell\theta_1^\top & \ell\theta_2^\top \\ 0 & \Lambda \end{bmatrix} \zeta + \begin{bmatrix} 0 \\ \ell \end{bmatrix} u \right) = A\Pi\zeta + bu \\ &= A\xi + bu \\ \eta &= \begin{bmatrix} \theta_1^\top & \theta_2^\top \end{bmatrix} \zeta = c^\top \Pi \zeta = c^\top \xi. \end{aligned} \tag{69}$$

The result follows from the same arguments in the proof of Lemma 1. □

7.5. *Proof of Lemma 5*

In (33), let

$$F := \begin{bmatrix} \Lambda + \ell\theta_1^\top & \ell\theta_2^\top \\ 0 & \Lambda \end{bmatrix}, \qquad g := \begin{bmatrix} 0 \\ \ell \end{bmatrix}. \tag{70}$$

By (Antsaklis and Michel, 2006, Ch. 3, Thm. 2.17), (F, g) is controllable if and only if the $2n$ rows of

$$\mathcal{H}(s) := (sI - F)^{-1} g \tag{71}$$

are linearly independent over the field of complex numbers, i.e., if and only if there exists no vector $w \in \mathbb{C}^{2n}$, $w \neq 0$, such that

$$w^\top \mathcal{H}(s) = 0, \qquad \text{for all } s. \tag{72}$$

By Lemma 4, $\mathcal{H}(s)$ can be equivalently seen as the transfer function of filters (31) combined with that of plant (29):

$$\mathcal{H}(s) = \begin{bmatrix} (sI-\Lambda)^{-1}\ell c^\top (sI-A)^{-1}b \\ (sI-\Lambda)^{-1}\ell \end{bmatrix}. \tag{73}$$

We now follow similar steps to (Sastry and Bodson, 1990, Thm. 2.7.3). Assume that there exists a vector $w \neq 0$ such that $w^\top \mathcal{H}(s) = 0$ for all s. Split $w = \mathrm{col}(w_1, w_2)$, with $w_1, w_2 \in \mathbb{C}^n$. Then, it holds that

$$\frac{\mathcal{N}_1(s)}{\mathcal{D}_\Lambda(s)} \frac{\mathcal{N}(s)}{\mathcal{D}(s)} + \frac{\mathcal{N}_2(s)}{\mathcal{D}_\Lambda(s)} = 0, \qquad \text{for all } s, \tag{74}$$

where $\mathcal{N}(s)$, $\mathcal{D}(s)$, $\mathcal{N}_1(s)$, $\mathcal{N}_2(s)$, and $\mathcal{D}_\Lambda(s)$ are polynomials of s such that $\frac{\mathcal{N}(s)}{\mathcal{D}(s)} = c^\top(sI-A)^{-1}b$, $\frac{\mathcal{N}_1(s)}{\mathcal{D}_\Lambda(s)} = w_1^\top(sI-\Lambda)^{-1}\ell$, and $\frac{\mathcal{N}_2(s)}{\mathcal{D}_\Lambda(s)} = w_2^\top(sI-\Lambda)^{-1}\ell$. For the rational function in (74) to be identically zero, it must hold that

$$\mathcal{N}_2(s) = -\mathcal{N}_1(s)\frac{\mathcal{N}(s)}{\mathcal{D}(s)}, \qquad \text{for all } s. \tag{75}$$

Note that $\mathcal{N}_1(s)$ and $\mathcal{N}_2(s)$ are at most of degree $n-1$, while $\mathcal{D}(s)$ has degree n. As a consequence, to ensure that the left- and the right-hand sides are equal, there must be n pole-zero cancellations in $\mathcal{N}_1(s)\frac{\mathcal{N}(s)}{\mathcal{D}(s)}$. From these considerations, it necessarily follows that there is at least one pole-zero cancellation in $\frac{\mathcal{N}(s)}{\mathcal{D}(s)}$. However, this property cannot be satisfied because $\mathcal{N}(s)$ and $\mathcal{D}(s)$ are coprime by Assumption 2, hence we have a contradiction. □

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