Data-Driven Stabilization of Continuous-Time LTI Systems from Noisy Input–Output Data*

Alessandro Bosso[a], Marco Borghesi[a], Andrea Iannelli[b], Bowen Yi[c], Giuseppe Notarstefano[a]

[a]*Department of Electrical, Electronic, and Information Engineering, University of Bologna, Bologna, Italy*
[b]*Institute for System Theory and Automatic Control, University of Stuttgart, Stuttgart, Germany*
[c]*Department of Electrical Engineering, Polytechnique Montréal and GERAD, Montréal, Canada*

Abstract

We present an approach to compute stabilizing controllers for continuous-time linear time-invariant systems directly from an input–output trajectory affected by process and measurement noise. The proposed output-feedback design combines (i) an observer of a non-minimal realization of the plant and (ii) a feedback law obtained from a linear matrix inequality (LMI) that depends solely on the available data. Under a suitable interval excitation condition and knowledge of a noise energy bound, the feasibility of the LMI is shown to be necessary and sufficient for stabilizing all non-minimal realizations consistent with the data. We further provide a condition for the feasibility of the LMI related to the signal-to-noise ratio, guidelines to compute the noise energy bound, and numerical simulations that illustrate the approach.

Keywords: Data-driven control, linear systems, linear matrix inequalities, uncertain systems

1. Introduction

Recent trends in automatic control highlight a growing reliance on data-based methods. Among them, the design of controllers directly from data—rooted in classical adaptive control (Ioannou and Sun, 2012)—has gained significant momentum. In particular, *data-driven control* (van Waarde et al., 2025) has become a central paradigm, inspired by the wave of results related to Willems et al.'s fundamental lemma (Willems et al., 2005) and fueled by the increasing availability of large datasets. This approach aims to map the available data into controllers via linear matrix inequalities (LMIs) or related optimization programs, leading to an end-to-end methodology that avoids any modeling or identification step.

In the discrete-time domain, two fundamental contributions to data-driven control are De Persis and Tesi (2020) and van Waarde et al. (2020), focusing on linear time-invariant (LTI) systems. These works proposed LMI formulations based respectively on data-based closed-loop system parameterizations and the data informativity framework. Later, data-driven LMIs particularly suited to noisy datasets have been developed from open-loop system parameterizations (Bisoffi et al., 2022; van Waarde et al., 2023a, 2025). Notably, these approaches enable the enforcement of design specifications (e.g., closed-loop stability and $\mathcal{H}_2$/$\mathcal{H}_\infty$ performance) for all systems consistent with the data. Further developments include techniques for linear quadratic regulation (Dörfler et al., 2023), partial model knowledge (Berberich et al., 2022), time-varying systems (Nortmann and Mylvaganam, 2023), nonlinear systems (Monshizadeh et al., 2025), systems subject to input saturations (Seuret and Tarbouriech, 2024), and output-feedback control based on the behavioral framework (van Waarde et al., 2023b) or non-minimal realizations constructed from shifted input–output data (Alsalti et al., 2025).

In the continuous-time setting, similar LMIs have been developed for data-driven state-feedback control (De Persis and Tesi, 2020; Eising and Cortes, 2024; Hu et al., 2025), also accounting for the presence of noise, but at the price of requiring state derivative measurements in the dataset. To circumvent this limitation, research efforts have been dedicated to achieving *derivative-free* state-feedback approaches (Rapisarda et al., 2023; Ohta and Rapisarda, 2024). Despite these advancements, data-driven control of continuous-time systems from input–output data remains far less developed. In this direction, Bosso et al. (2025) and Bosso et al. (2026) proposed a derivative-free framework to design output-feedback controllers with a filter of the input–output data acting as an observer of a non-minimal realization of the plant. While this approach achieves stabilization and output regulation in the noise-free setting, to the best of the authors' knowledge, no method can yet handle noisy input–output data. In particular, the challenge is to derive computationally tractable controller design conditions that account for all systems consistent with the dataset.

Motivated by this gap, this paper proposes an approach for data-driven stabilization of a class of multi-input multi-output (MIMO) systems subject to process and measurement noise

*The research leading to these results has received funding from the European Union's Horizon Europe research and innovation program under the Marie Skłodowska-Curie Grant Agreement No. 101104404 - IMPACT4Mech.

Email addresses: alessandro.bosso@unibo.it (Alessandro Bosso), m.borghesi@unibo.it (Marco Borghesi), andrea.iannelli@ist.uni-stuttgart.de (Andrea Iannelli), bowen.yi@polymtl.ca (Bowen Yi), giuseppe.notarstefano@unibo.it (Giuseppe Notarstefano)

that generalizes the results of Bosso et al. (2026). The proposed control design comprises a filter of the input–output signals and a linear feedback law depending on the filter states. The gains of the feedback law are computed with an LMI obtained by postprocessing the offline data through the filter dynamics. In contrast with Bosso et al. (2026), which relies on a closed-loop system parameterization similar to De Persis and Tesi (2020), we adopt an open-loop parameterization that leverages the quadratic matrix inequality (QMI) formalism of van Waarde et al. (2023a). Also, different from Bosso et al. (2026), the proposed robust LMI is constructed from an integral functional of the available signals instead of collections of sample batches. The resulting algorithm exploits information contained in the entire trajectory and ensures that the decision variables do not scale with the experiment duration, thus enabling the treatment of large datasets. Under an interval excitation condition and assuming the knowledge of a noise energy bound, we use a matrix version of the S-lemma to prove that the feasibility of the LMI is necessary and sufficient for stabilizing all non-minimal realizations consistent with the data, characterized by a bounded matrix ellipsoid. We then relate the feasibility of the LMI to the signal-to-noise ratio of the dataset. Finally, we show a method to compute the noise energy based on assumptions on the noise and the tuning of the filters.

The paper is organized as follows. In Section 2, we introduce the considered control problem. In Section 3, we illustrate our approach and its theoretical guarantees. In Section 4, we provide a sufficient condition for LMI feasibility, while Section 5 shows how to compute the noise energy bound. Finally, Section 6 presents two numerical examples and Section 7 concludes the paper. Further details and the technical proofs are left in the Appendix.

Notation

We use $\mathbb{R}$ and $\mathbb{C}$ to denote the sets of real and complex numbers, respectively, while i is the imaginary unit. I_j denotes the identity matrix of dimension j, while 0_j and $0_{j\times k}$ denote zero matrices of dimensions $j \times j$ and $j \times k$. $\otimes$ is the Kronecker product between matrices. We denote by $\sigma(R)$ the spectrum of a square matrix R. Given a symmetric matrix S, $\lambda_{\min}(S)$ and $\lambda_{\max}(S)$ denote its minimum and maximum eigenvalues, while $S > 0$ (resp., $S \geq 0$) denotes that it is positive definite (resp., positive semidefinite). Similarly, < 0 and ≤ 0 are used for negative definite and semidefinite matrices. For symmetric matrices S_1 and S_2, we write $S_1 \leq S_2$ if $S_1 - S_2 \leq 0$. Given a signal $f(\cdot) : [0,T] \to \mathbb{R}^l$, its finite-horizon $\mathcal{L}_2[0,T]$ norm is $\|f(\cdot)\|_{2,[0,T]} := \left(\int_0^T |f(\tau)|^2 \mathrm{d}\tau\right)^{1/2}$. Finally, the $\mathcal{H}_\infty$ norm of a transfer function matrix $\mathcal{G}(s)$ is denoted by $\|\mathcal{G}\|_\infty$.

2. Problem Statement

We consider continuous-time linear time-invariant systems described by the following input–output model:

$$\mathcal{D}\left(\frac{\mathrm{d}}{\mathrm{d}t}\right)(y - v) = \mathcal{N}\left(\frac{\mathrm{d}}{\mathrm{d}t}\right)u + \mathcal{N}_w\left(\frac{\mathrm{d}}{\mathrm{d}t}\right)w, \tag{1}$$

where $u \in \mathbb{R}^m$ is the control input, $y \in \mathbb{R}^p$ is the measured output, $w \in \mathbb{R}^q$ is the process noise, and $v \in \mathbb{R}^p$ is the measurement noise, while, for some known positive integer n that we denote as *order* of the differential equation (1),

$$\begin{aligned}
\mathcal{D}(\xi) &:= I_p\xi^n + A_{n-1}\xi^{n-1} + \ldots + A_1\xi + A_0\\
\mathcal{N}(\xi) &:= B_{n-1}\xi^{n-1} + \ldots + B_1\xi + B_0\\
\mathcal{N}_w(\xi) &:= E_{n-1}\xi^{n-1} + \ldots + E_1\xi + E_0
\end{aligned} \tag{2}$$

are polynomial matrices with unknown matrix coefficients $A_i \in \mathbb{R}^{p\times p}$, $B_i \in \mathbb{R}^{p\times m}$, $i \in \{0, \ldots, n-1\}$ and known matrix coefficients $E_i \in \mathbb{R}^{p\times q}$, $i \in \{0, \ldots, n-1\}$. Note that the solutions of the differential equation (1) are intended in the weak sense as per Polderman and Willems (1998, Def. 2.3.7), so that the signals are not required to be differentiable but only locally integrable.

We represent the input–output model (1) with the following state-space realization, having state $x \in \mathbb{R}^{np}$:

$$\begin{aligned}
\dot{x} &= \underbrace{\left[\begin{array}{cccc|c} 0_p & \cdots & & 0_p & -A_0\\ I_p & & & & -A_1\\ & \ddots & & & \vdots\\ & & & I_p & -A_{n-1}\end{array}\right]}_{A} x + \underbrace{\begin{bmatrix} B_0\\ B_1\\ \vdots\\ B_{n-1}\end{bmatrix}}_{B} u + \underbrace{\begin{bmatrix} E_0\\ E_1\\ \vdots\\ E_{n-1}\end{bmatrix}}_{E} w\\
y &= \underbrace{\left[\begin{array}{ccc|c} 0_p & \cdots & 0_p & I_p\end{array}\right]}_{C} x + v,
\end{aligned} \tag{3}$$

where the pair (C, A) is observable. Further details on the realization (3) are provided in the Appendix. We also assume the following property.

Assumption 1. *The pair (A, B) is controllable.*

Note that Assumption 1 is equivalent to requiring that the polynomial matrices $\mathcal{D}$ and $\mathcal{N}$ are left coprime, see Antsaklis and Michel (2006, Ch. 7).

Suppose that an experiment is performed on system (3) over the time interval $[0, T]$, with $T > 0$. Specifically, given an unknown initial condition $x(0) \in \mathbb{R}^{np}$, an input trajectory $u(\cdot) : [0, T] \to \mathbb{R}^m$, and unknown noise signals $w(\cdot) : [0, T] \to \mathbb{R}^q$, $v(\cdot) : [0, T] \to \mathbb{R}^p$, let the corresponding state and output trajectories be $x(\cdot) : [0, T] \to \mathbb{R}^{np}$ and $y(\cdot) : [0, T] \to \mathbb{R}^p$, with $x(\cdot)$ an absolutely continuous function. Then, suppose that the continuous-time dataset $\{(t, u(t), y(t)) : t \in [0, T]\}$ is collected during the experiment.

Our goal is to use such a dataset in an offline procedure that designs a stabilizing controller for system (1) (equivalently, system (3)) without any prior knowledge of A_i, B_i, $i \in \{1, \ldots, n-1\}$ except for the order n. In particular, we aim to compute the matrices A_c, B_c, C_c, and D_c of a dynamic controller of the form

$$\begin{aligned}
\dot{x}_\mathrm{c} &= A_\mathrm{c}x_\mathrm{c} + B_\mathrm{c}y\\
u &= C_\mathrm{c}x_\mathrm{c} + D_\mathrm{c}y,
\end{aligned} \tag{4}$$

with controller state $x_\mathrm{c} \in \mathbb{R}^\mu$, so that the following matrix, obtained from the interconnection of (3) and (4), is Hurwitz:

$$\begin{bmatrix} A + BD_\mathrm{c}C & BC_\mathrm{c}\\ B_\mathrm{c}C & A_\mathrm{c}\end{bmatrix}. \tag{5}$$

3. Data-Driven Stabilization from Noisy Input–Output Data

We now illustrate the proposed data-driven stabilization approach, whose implementation is provided in Algorithm 1. The central idea is to build a non-minimal realization of the plant (Section 3.1) that has a structure amenable to observer design. This allows us to split the dynamic controller into an observer of the non-minimal realization and a feedback law depending on the observer states (Section 3.2). In particular, the feedback gain is designed to quadratically stabilize all realizations consistent with the data (Section 3.3) by employing a data-based LMI derived from the matrix S-lemma (van Waarde et al., 2023a, Thm. 4.10) (Section 3.4).

3.1. Non-Minimal Realization for Data-Driven Control

Inspired by classical results in adaptive control (Narendra and Annaswamy, 1989, Ch. 4) and following the state-space framework developed in Bosso et al. (2026), we now introduce a non-minimal realization of the transfer function matrix $\mathcal{D}^{-1}(s)\mathcal{N}(s)$, characterizing the noise-free input–output behavior of (1). Let (Λ, Γ) be a controllable pair as described in Algorithm 1, and let

$$\mathcal{D}_\lambda(\xi) := \xi^n + \lambda_{n-1}\xi^{n-1} + \ldots + \lambda_1\xi + \lambda_0 \tag{11}$$

be the characteristic polynomial of Λ, having n distinct roots with negative real part. As discussed further in Section 3.2, Λ is used to tune the dynamics of the observer employed in the proposed control architecture. Then, define $F \in \mathbb{R}^{\mu\times\mu}$, $G \in \mathbb{R}^{\mu\times m}$, $L \in \mathbb{R}^{\mu\times p}$ as in (7), with $\mu := n(p+m)$ that represents, as shown below, the dimension of the non-minimal realization.

The following result will be used throughout the paper. The proof is based on Bosso et al. (2026) and can be found in the Appendix.

Lemma 1. *Under Assumption 1 and given F, G, and L as in (7), there exist full-rank matrices $\Pi \in \mathbb{R}^{np\times\mu}$, $H \in \mathbb{R}^{p\times\mu}$ that satisfy*

$$\Pi(F + LH) = A\Pi, \quad \Pi G = B, \quad H = C\Pi, \tag{12}$$

and, in addition, ensure the following properties:

1. *$(F + LH, G)$ is controllable.*
2. *$A - \Pi LC$ has the form*

$$\tilde{\Lambda} := A - \Pi LC = \left[\begin{array}{cccc|c} 0_p & \cdots & & 0_p & -\lambda_0 I_p \\ I_p & & & & -\lambda_1 I_p \\ & \ddots & & & \vdots \\ & & & I_p & -\lambda_{n-1} I_p \end{array}\right]. \tag{13}$$

Using Π and H from Lemma 1, we obtain from Bosso et al. (2026, Lemma 1) that the following dynamical system

$$\begin{aligned} \dot{z} &= (F + LH)z + Gu \\ y &= Hz, \end{aligned} \tag{14}$$

with $z \in \mathbb{R}^\mu$, is a controllable and detectable (but not observable) non-minimal realization of $\mathcal{D}^{-1}(s)\mathcal{N}(s)$. In particular, the controllable and observable subsystem of (14), parameterized via $x = \Pi z$, obeys $\dot{x} = Ax + Bu$, $y = Cx$.

3.2. Observer-Based Control Design

The special structure of (14), where F is Hurwitz by construction (see Algorithm 1) and Hz can be replaced by y in the differential equation, allows us to design an observer of the plant (3) in the coordinates of the non-minimal realization, without any knowledge of the parameters in A and B. In particular, define the following dynamical system:

$$\dot{\hat{z}} = F\hat{z} + Gu + Ly, \tag{15}$$

Algorithm 1 Data-Driven Stabilization from Noisy I–O Data

Initialization

Measured dataset:

$$\{(t, u(t), y(t)) : t \in [0, T]\}. \tag{6}$$

Controller state dimension: $\mu := n(p + m)$.

Tuning: $\Lambda \in \mathbb{R}^{n\times n}$ Hurwitz with n distinct eigenvalues; $\Gamma \in \mathbb{R}^n$ such that (Λ, Γ) is controllable; $\Delta \in \mathbb{R}^{p\times p}$, $\Delta = \Delta^\top \geq 0$.

Filter matrices:

$$F := I_{p+m} \otimes \Lambda, \quad G := \begin{bmatrix} 0_{np\times m} \\ I_m \otimes \Gamma \end{bmatrix}, \quad L := \begin{bmatrix} I_p \otimes \Gamma \\ 0_{nm\times p} \end{bmatrix}. \tag{7}$$

Filtering

Filter signals: using (6), compute for all $t \in [0, T]$:

$$\zeta(t) := \begin{bmatrix} \chi(t) \\ \hat{z}(t) \end{bmatrix} := \begin{bmatrix} e^{\Lambda t}\Gamma \\ \int_0^t e^{F(t-\tau)}(Gu(\tau) + Ly(\tau))\mathrm{d}\tau \end{bmatrix}. \tag{8}$$

Controller Design

LMI: find $P \in \mathbb{R}^{\mu\times\mu}$ and $Q \in \mathbb{R}^{m\times\mu}$ satisfying the linear matrix inequality $(\mathcal{S})$ reported below.

Control gain:

$$K := QP^{-1}. \tag{9}$$

Controller matrices: in (4), let:

$$A_c = F + GK, \quad B_c = L, \quad C_c = K, \quad D_c = 0_{m\times p}. \tag{10}$$

$$\int_0^T \begin{bmatrix} Ly(\tau) \\ -\zeta(\tau) \end{bmatrix}\begin{bmatrix} Ly(\tau) \\ -\zeta(\tau) \end{bmatrix}^\top \mathrm{d}\tau - \left[\begin{array}{c|cc} L\Delta L^\top + FP + PF^\top + GQ + Q^\top G^\top & 0_{\mu\times n} & P \\ \hline \begin{matrix} 0_{n\times\mu} \\ P \end{matrix} & \multicolumn{2}{c}{0_{n+\mu}} \end{array}\right] \succ 0, \qquad P = P^\top \succ 0. \tag{$\mathcal{S}$}$$

having the same structure of (14), with user-defined matrices F, G, L as per Algorithm 1. Then, define the estimation error

$$\tilde{x} := x - \Pi\hat{z}. \tag{16}$$

Using (3), (12), (15), (16), the observer dynamics become

$$\begin{aligned}\dot{\hat{z}} &= F\hat{z} + Gu + L(Cx + v)\\ &= F\hat{z} + Gu + LC(\tilde{x} + \Pi\hat{z}) + Lv\\ &= (F + LH)\hat{z} + Gu + LC\tilde{x} + Lv,\end{aligned} \tag{17}$$

and then we obtain the following estimation error dynamics:

$$\begin{aligned}\dot{\tilde{x}} &= A(\tilde{x} + \Pi\hat{z}) + Bu + Ew\\ &\quad - \Pi((F + LH)\hat{z} + Gu + LC\tilde{x} + Lv)\\ &= (A - \Pi LC)\tilde{x} + Ew - \Pi Lv\\ &= \tilde{\Lambda}\tilde{x} + Ew - \Pi Lv.\end{aligned} \tag{18}$$

The interconnection of (3) and (15) can be rewritten as:

$$\begin{bmatrix}\dot{\tilde{x}}\\ \dot{\hat{z}}\end{bmatrix} = \begin{bmatrix}\tilde{\Lambda} & 0_{np\times\mu}\\ LC & F + LH\end{bmatrix}\begin{bmatrix}\tilde{x}\\ \hat{z}\end{bmatrix} + \begin{bmatrix}0_{np\times m}\\ G\end{bmatrix}u + \begin{bmatrix}E & -\Pi L\\ 0_{\mu\times q} & L\end{bmatrix}\begin{bmatrix}w\\ v\end{bmatrix}. \tag{19}$$

Since $\tilde{\Lambda}$ in (13) is Hurwitz and the pair $(F + LH, G)$ is controllable, the desired controller (4) is obtained by combining the observer (15) and the feedback law

$$u = K\hat{z}, \tag{20}$$

where the gain $K \in \mathbb{R}^{m\times\mu}$ is chosen so that $F + LH + GK$ is Hurwitz. Since H depends on the unknown plant parameters (see (12)), we cannot design K using solely the prior knowledge of $(F + LH, G)$. Therefore, in the following, we compute K using the input–output trajectory (6).

Remark 1. *The proposed approach is the data-driven counterpart of classical output-feedback control design based on Luenberger observers; see Bosso et al. (2026, Rem. 3) for further details. In this context, due to the structure of $\tilde{\Lambda}$ in* (13)*, Λ determines the transient behavior of the observer estimation error dynamics.*

3.3. Non-Minimal Realizations Consistent with the Dataset

Given the trajectories $u(\cdot)$ and $y(\cdot)$ over the interval $[0, T]$, we can simulate offline the observer (15), which acts as a linear filter of the data. Without loss of generality and for simplicity, we consider $\hat{z}(0) = 0$, so that we obtain $\hat{z}(\cdot) : [0, T] \to \mathbb{R}^\mu$ as defined in (8). It follows that the dataset and the simulated observer trajectory $\hat{z}(\cdot)$ satisfy (19) for almost all $t \in [0, T]$. Also, for all $t \in [0, T]$, the error $\tilde{x}(t) = x(t) - \Pi\hat{z}(t)$ can be written explicitly using (18) and $\tilde{x}(0) = x(0)$:

$$\tilde{x}(t) = e^{\tilde{\Lambda}t}x(0) + \int_0^t e^{\tilde{\Lambda}(t-\tau)}(Ew(\tau) - \Pi Lv(\tau))\mathrm{d}\tau. \tag{21}$$

We now exploit a technical result adapted from Bosso et al. (2026, Lemma 4) to rewrite $e^{\tilde{\Lambda}t}x(0)$ in (21) in a convenient form. The proof is reported in the Appendix.

Lemma 2. *For any $x(0) \in \mathbb{R}^{np}$ and any (Λ, Γ) as defined in Algorithm 1, there exists $H_0 \in \mathbb{R}^{p\times n}$ such that, for all $t \in [0, T]$, $Ce^{\tilde{\Lambda}t}x(0) = H_0\chi(t)$, where $\chi(t)$ is given in* (8).

Combining $y(t) = C\tilde{x}(t) + H\hat{z}(t) + v(t)$ with (21) and Lemma 2 we obtain, for all $t \in [0, T]$,

$$y(t) = \Theta^\star\zeta(t) + d(t), \tag{22}$$

where $\zeta(t)$ is defined in (8),

$$\Theta^\star := \begin{bmatrix}H_0 & H\end{bmatrix} \in \mathbb{R}^{p\times(n+\mu)}, \tag{23}$$

denotes the parameters derived from the unknown matrices A and B and the initial condition $x(0)$, while, for all $t \in [0, T]$,

$$d(t) := v(t) + C\int_0^t e^{\tilde{\Lambda}(t-\tau)}(Ew(\tau) - \Pi Lv(\tau))\mathrm{d}\tau. \tag{24}$$

Note that $y(t)$ and $\zeta(t)$ are available for measurement, while $\Theta^\star$ and $d(t)$ are unknown. In particular, to represent the lack of knowledge of $\Theta^\star$, we now describe the set of all matrices $\Theta \in \mathbb{R}^{p\times(n+\mu)}$ that are consistent with the dataset (6). To obtain a set with suitable regularity properties, we impose the following assumptions on the signals appearing in (22).

Assumption 2. *The matrix Δ in Algorithm 1 is such that*

$$\int_0^T d(\tau)d^\top(\tau)\mathrm{d}\tau \preceq \Delta. \tag{25}$$

Assumption 3. *The following excitation condition holds:*

$$Z := \int_0^T \zeta(\tau)\zeta^\top(\tau)\mathrm{d}\tau \succ 0. \tag{26}$$

Remark 2. *Similar to the state-feedback scenario of Eising and Cortes (2024), Assumption 2 allows us to represent the set of parameters Θ as the solutions of a QMI and, then, exploit the results of van Waarde et al. (2023a) for data-driven stabilization. In Section 5, we show how to compute Δ satisfying* (25) *when certain prior knowledge about the noise signals $w(\cdot)$ and $v(\cdot)$ is available.*

Remark 3. *Assumption 3 corresponds to the full-rank conditions of the data-driven literature (De Persis and Tesi, 2020; Bisoffi et al., 2022; van Waarde et al., 2023b). Although* (26) *is not necessary for introducing the non-strict LMI of van Waarde et al. (2023a, Cor. 4.13), it provides improved theoretical guarantees and leads to the strict LMI of van Waarde et al. (2023a, Thm. 4.10) which, as argued there, may be preferred in terms of numerical reliability.*

For compactness of notation, define

$$\left[\begin{array}{c|c} Y & X^\top\\ \hline X & Z\end{array}\right] := \int_0^T \left[\begin{array}{c} y(\tau)\\ \hline -\zeta(\tau)\end{array}\right]\left[\begin{array}{c} y(\tau)\\ \hline -\zeta(\tau)\end{array}\right]^\top \mathrm{d}\tau \succeq 0, \tag{27}$$

where the lines indicate the corresponding blocks, so that $Y \in \mathbb{R}^{p\times p}$, $X \in \mathbb{R}^{(n+\mu)\times p}$, and $Z \in \mathbb{R}^{(n+\mu)\times(n+\mu)}$.

Then, replace $d(t)$ with $y(t) - \Theta\zeta(t)$ in (25) to obtain the set of all non-minimal realizations consistent with (6):

$$\mathcal{E} := \left\{\Theta \in \mathbb{R}^{p\times(\mu+n)} : \begin{bmatrix} I_p & \Theta \end{bmatrix} N \begin{bmatrix} I_p \\ \Theta^\top \end{bmatrix} \succeq 0 \right\}, \tag{28}$$

where

$$N := \begin{bmatrix} \Delta - Y & -X^\top \\ -X & -Z \end{bmatrix}. \tag{29}$$

3.4. Data-Driven Stabilization via the Matrix S-Lemma

To solve the stabilization problem of Section 2, we need to find a gain K for the feedback law (20) that stabilizes all systems contained in the uncertainty set $\mathcal{E}$ of (28). More precisely, we want to find a pair of matrices $P \in \mathbb{R}^{\mu\times\mu}$ and $K \in \mathbb{R}^{m\times\mu}$ such that $P = P^\top \succ 0$ and, for all $\Theta \in \mathcal{E}$,

$$\left(F + L\Theta \begin{bmatrix} 0_{n\times\mu} \\ I_\mu \end{bmatrix} + GK\right) P + P\left(F + L\Theta \begin{bmatrix} 0_{n\times\mu} \\ I_\mu \end{bmatrix} + GK\right)^\top \prec 0, \tag{30}$$

which implies that (5) is Hurwitz by choosing the controller (15), (20). We now translate this property into an inclusion of sets described by QMIs. Using (30), the set of all non-minimal realizations stabilized by a pair (P, K) is

$$\mathcal{C}(P, K) := \left\{\Theta \in \mathbb{R}^{p\times(\mu+n)} : \begin{bmatrix} I_\mu \\ (L\Theta)^\top \end{bmatrix}^\top M(P, K) \begin{bmatrix} I_\mu \\ (L\Theta)^\top \end{bmatrix} \succ 0 \right\}, \tag{31}$$

where

$$M(P, K) := -\left[\begin{array}{c|cc} FP + PF^\top + GKP + PK^\top G^\top & 0_{\mu\times n} & P \\ \hline \begin{matrix} 0_{n\times\mu} \\ P \end{matrix} & \multicolumn{2}{c}{0_{n+\mu}} \end{array}\right]. \tag{32}$$

Then, the property that we want to impose is the following:

$$\mathcal{E} \subseteq \mathcal{C}(P, K). \tag{33}$$

The next statement, which relates (33) with the feasibility of the LMI ($\mathcal{S}$) used in Algorithm 1, is the main result of this work. Its proof is based on the strict matrix S-lemma found in van Waarde et al. (2023a, Thm. 4.10).

Theorem 1. *Consider Algorithm 1 and let Assumptions 1, 2, and 3 hold. Then, the following statements are equivalent:*

1. *There exist $P \in \mathbb{R}^{\mu\times\mu}$ and $K \in \mathbb{R}^{m\times\mu}$, with $P = P^\top \succ 0$, such that $\mathcal{E} \subseteq \mathcal{C}(P, K)$.*
2. *There exist $P \in \mathbb{R}^{\mu\times\mu}$ and $Q \in \mathbb{R}^{m\times\mu}$ such that the LMI ($\mathcal{S}$) is satisfied.*

Moreover, if ($\mathcal{S}$) is feasible, then $K = QP^{-1}$ is such that $F + LH + GK$ is Hurwitz and, thus, the controller (4) *with gains given in* (10) *solves the data-driven stabilization problem of Section 2.*

Proof: Define the following sets:

$$\begin{aligned} \mathcal{E}_\Psi &:= \left\{\Psi \in \mathbb{R}^{\mu\times(n+\mu)} : \begin{bmatrix} I_\mu \\ \Psi^\top \end{bmatrix}^\top N_\Psi \begin{bmatrix} I_\mu \\ \Psi^\top \end{bmatrix} \succeq 0 \right\} \\ \mathcal{C}_\Psi(P, K) &:= \left\{\Psi \in \mathbb{R}^{\mu\times(n+\mu)} : \begin{bmatrix} I_\mu \\ \Psi^\top \end{bmatrix}^\top M(P, K) \begin{bmatrix} I_\mu \\ \Psi^\top \end{bmatrix} \succ 0 \right\}, \end{aligned} \tag{34}$$

where

$$N_\Psi := \begin{bmatrix} L & \\ & I_{n+\mu} \end{bmatrix} N \begin{bmatrix} L^\top & \\ & I_{n+\mu} \end{bmatrix} = \begin{bmatrix} L(\Delta - Y)L^\top & -LX^\top \\ -XL^\top & -Z \end{bmatrix}. \tag{35}$$

By van Waarde et al. (2023a, Thm. 3.4), $L\mathcal{E} := \{L\Theta : \Theta \in \mathcal{E}\} = \mathcal{E}_\Psi$ because $Z \succ 0$ by Assumption 3. Furthermore, $L\mathcal{C}(P, K) := \{L\Theta : \Theta \in \mathcal{C}(P, K)\} \subseteq \mathcal{C}_\Psi(P, K)$ because, for any $\Theta \in \mathcal{C}(P, K)$, $\Psi = L\Theta \in \mathcal{C}_\Psi(P, K)$.

We prove the equivalence of the statement using van Waarde et al. (2023a, Thm. 4.10). To verify its assumptions, we inspect the matrix N_Ψ:

- The $(2, 2)$ block is negative definite as $Z \succ 0$.
- $L(\Delta - Y)L^\top - LX^\top Z^{-1} XL^\top \succeq 0$ because $\mathcal{E}_\Psi$ is nonempty, see van Waarde et al. (2023a, Eq. (3.4)).
- $\ker Z \subseteq \ker LX^\top$ because Z has full rank.

We are ready to prove both directions of the equivalence.

1 $\implies$ **2**: Let $P = P^\top \succ 0$ and K satisfy (33). It holds that

$$\mathcal{E}_\Psi = L\mathcal{E} \subseteq L\mathcal{C}(P, K) \subseteq \mathcal{C}_\Psi(P, K). \tag{36}$$

From van Waarde et al. (2023a, Thm. 4.10), $\mathcal{E}_\Psi \subseteq \mathcal{C}_\Psi(P, K)$ holds only if there exists $\alpha \geq 0$ such that

$$M(P, K) - \alpha N_\Psi \succ 0. \tag{37}$$

Note that, by construction, $M(P, K)$ does not have full rank, thus the feasibility of (37) implies $\alpha > 0$. As a consequence, divide (37) by α and redefine $\alpha^{-1}P$ and $\alpha^{-1}PK$ as P and Q to obtain the LMI ($\mathcal{S}$).

2 $\implies$ **1**: Let $P = P^\top \succ 0$ and Q satisfy ($\mathcal{S}$). If we define $K = QP^{-1}$, we obtain that (37) is satisfied with $\alpha = 1$. Then, by van Waarde et al. (2023a, Thm. 4.10), $\mathcal{E}_\Psi \subseteq \mathcal{C}_\Psi(P, K)$. From the fact that $\mathcal{E}_\Psi = L\mathcal{E}$, this inclusion corresponds to saying that if $\Theta \in \mathcal{E}$, then $L\Theta \in \mathcal{C}_\Psi(P, K)$, i.e., it satisfies (30). It follows that $\Theta \in \mathcal{C}(P, K)$, which implies (33). □

4. Feasibility of the LMI

We now exploit Theorem 1 to provide a sufficient condition based on data such that the LMI ($\mathcal{S}$) is feasible.

Under Assumption 3, $\mathcal{E}$ can be rewritten as a bounded matrix ellipsoid. By defining $\tilde{\Theta} := \Theta - \hat{\Theta}$, where

$$\hat{\Theta} := -X^\top Z^{-1} \tag{38}$$

is the *least-squares estimate* of $\Theta^\star$, we can change the coordinates in the QMI of (28) to obtain

$$\begin{bmatrix} I_p \\ \Theta^\top \end{bmatrix}^\top N \begin{bmatrix} I_p \\ \Theta^\top \end{bmatrix} = \begin{bmatrix} I_p \\ \tilde{\Theta}^\top \end{bmatrix}^\top \begin{bmatrix} S_N & 0_{p\times(n+\mu)} \\ 0_{(n+\mu)\times p} & -Z \end{bmatrix} \begin{bmatrix} I_p \\ \tilde{\Theta}^\top \end{bmatrix}, \tag{39}$$

where $S_N := \Delta - Y + X^\top Z^{-1} X$ is the Schur complement of the $(2, 2)$ block of N in (29). Thus, (28) becomes

$$\mathcal{E} = \left\{\Theta \in \mathbb{R}^{p\times(n+\mu)} : (\Theta - \hat{\Theta}) Z (\Theta - \hat{\Theta})^\top \preceq S_N \right\}. \tag{40}$$

From (27) and $Z > 0$, it holds from the Schur complement that $Y - X^\top Z^{-1} X \geq 0$. As a consequence,

$$\mathcal{E} \subseteq \hat{\mathcal{E}} := \{\Theta \in \mathbb{R}^{p\times(n+\mu)} : (\Theta - \hat{\Theta})Z(\Theta - \hat{\Theta})^\top \leq \Delta\}. \tag{41}$$

Exploiting the fact that, for any $\Theta \in \hat{\mathcal{E}}$, $\lambda_{\min}(Z)(\Theta - \hat{\Theta})(\Theta - \hat{\Theta})^\top \leq (\Theta - \hat{\Theta})Z(\Theta - \hat{\Theta})^\top \leq \Delta$, we obtain the bound

$$|\Theta - \hat{\Theta}|^2 \leq \lambda_{\max}(\Delta)/\lambda_{\min}(Z) =: \rho, \tag{42}$$

where, for $\rho > 0$, $\mathrm{SNR} := \rho^{-1}$ is the worst-case *signal-to-noise ratio* according to Assumptions 2 and 3. Note that $\Theta^\star \in \mathcal{E}$, thus it follows from the triangle inequality that

$$|\Theta - \Theta^\star| = |(\Theta - \hat{\Theta}) - (\Theta^\star - \hat{\Theta})| \leq 2\sqrt{\rho}. \tag{43}$$

From Theorem 1, we obtain that $(\mathcal{S})$ is feasible whenever ρ is sufficiently small, as stated in the following corollary. The proof can be found in the Appendix.

Corollary 1. *Let Assumption 1 hold. Then, there exists $\rho^\star > 0$ such that, for any dataset satisfying Assumptions 2 and 3 with $\rho \in [0, \rho^\star]$, the LMI $(\mathcal{S})$ is feasible.*

5. Remarks on the Noise Bound Computation

Note that the bound Δ in Assumption 2 depends both on the noise signals $w(\cdot)$, $v(\cdot)$ and the parameters of the plant (3) and the observer (15). Specifically, $d(t)$ in (24) can be split as $d(t) = d_w(t) + d_v(t)$, where $d_w(t)$ and $d_v(t)$ are the outputs of the following dynamical systems:

$$\dot{\eta}_w = \tilde{\Lambda}\eta_w + Ew, \qquad d_w = C\eta_w \tag{44}$$

$$\dot{\eta}_v = \tilde{\Lambda}\eta_v - \Pi L v, \qquad d_v = C\eta_v + v, \tag{45}$$

with zero initial conditions.

It is convenient to report the transfer function matrices of these systems:

$$\mathcal{G}_w(s) := \mathcal{D}_\lambda^{-1}(s)\mathcal{N}_w(s), \qquad \mathcal{G}_v(s) := \mathcal{D}_\lambda^{-1}(s)\mathcal{D}(s), \tag{46}$$

where $\mathcal{D}_\lambda$ is given in (11). $\mathcal{G}_w(s)$ and $\mathcal{G}_v(s)$ follow from similar computations to those in the Appendix after noticing that, due to (3) and (13), $\Pi L = [\lambda_0 I_p - A_0^\top \ \ldots \ \lambda_{n-1} I_p - A_{n-1}^\top]^\top$. Note that only $\mathcal{G}_v(s)$ depends on unknown parameters.

Using the properties $d(t)d(t)^\top \leq |d(t)|^2 I_p$ and $\|d(\cdot)\|_{2,[0,T]} \leq \|d_w(\cdot)\|_{2,[0,T]} + \|d_v(\cdot)\|_{2,[0,T]}$, we obtain

$$\int_0^T d(\tau)d^\top(\tau)\mathrm{d}\tau \leq I_p(\|d_w(\cdot)\|_{2,[0,T]} + \|d_v(\cdot)\|_{2,[0,T]})^2. \tag{47}$$

Thus, Δ in (25) can be computed using (47) if upper bounds of $\|d_w(\cdot)\|_{2,[0,T]}$ and $\|d_v(\cdot)\|_{2,[0,T]}$ are known. In the following, we show how to compute them under certain prior knowledge of $w(\cdot)$ and $v(\cdot)$ and suitable properties of $\mathcal{G}_w(s)$ and $\mathcal{G}_v(s)$.

5.1. Bound on the Filtered Process Noise

Suppose that a scalar $\delta_w \geq 0$ is known such that $\|w(\cdot)\|_{2,[0,T]}^2 \leq \delta_w$. Then, a bound on $\|d_w(\cdot)\|_{\mathcal{L}_2,[0,T]}$ is obtained by finding an upper bound $\gamma > 0$ of the $\mathcal{L}_2[0,T]$ gain of system (44), which ensures:

$$\|d_w(\cdot)\|_{2,[0,T]} \leq \gamma\|w(\cdot)\|_{2,[0,T]} \leq \gamma\sqrt{\delta_w}. \tag{48}$$

By Green and Limebeer (1995, Thm. 3.7.4), γ is such that the following differential Riccati equation:

$$\dot{W} = -\tilde{\Lambda}^\top W - W\tilde{\Lambda} - \gamma^{-2}WEE^\top W - C^\top C, \quad W(T) = 0_{np}, \tag{49}$$

has a solution over $[0,T]$. Since all matrices in (44) are known, (49) can be used to test γ. Let $\gamma_\infty > 0$ be such that $\|\mathcal{G}_w\|_\infty < \gamma_\infty$. Then, a conservative choice for γ is γ_∞, because by Green and Limebeer (1995, Lemma 3.7.7) (49) with $\gamma = \gamma_\infty$ has a solution over $[0,T]$ for all finite T. Otherwise, a tight bound can be found by searching in the interval $[0, \gamma_\infty]$ for $\gamma > 0$ as small as possible such that (49) has a solution over $[0,T]$.

5.2. Bound on the Filtered Measurement Noise

In this scenario, we cannot use the same arguments as before because ΠL in (45) depends on the unknown parameters in A. However, in the special case of single-output systems ($p = 1$), we obtain the following result, which can be imposed without the knowledge of A by making the observer dynamics sufficiently fast with the tuning gain Λ. The proof is given in the Appendix.

Proposition 1. *Suppose that $p = 1$ and let all eigenvalues of Λ be real and such that*

$$\min_{\varsigma\in\sigma(\Lambda)} |\varsigma| \geq \max_{\varphi\in\sigma(A)} |\varphi|. \tag{50}$$

Then, it holds that $\|\mathcal{G}_v\|_\infty = 1$.

Under the assumptions of Proposition 1, if a scalar $\delta_v \geq 0$ is known such that $\|v(\cdot)\|_{2,[0,T]}^2 \leq \delta_v$, we obtain

$$\|d_v(\cdot)\|_{2,[0,T]} \leq \sqrt{\delta_v}. \tag{51}$$

Future work will investigate the problem of computing a bound for $\|d_v(\cdot)\|_{2,[0,T]}$ in the multi-output scenario ($p > 1$).

6. Numerical Results

We now apply Algorithm 1 to some case studies. The code has been developed in MATLAB using YALMIP (Löfberg, 2004) and MOSEK (ApS, 2024) and is available on Zenodo (Bosso, 2026) and GitHub[1].

[1] https://github.com/IMPACT4Mech/continuous-time_stabilization_from_noisy_data

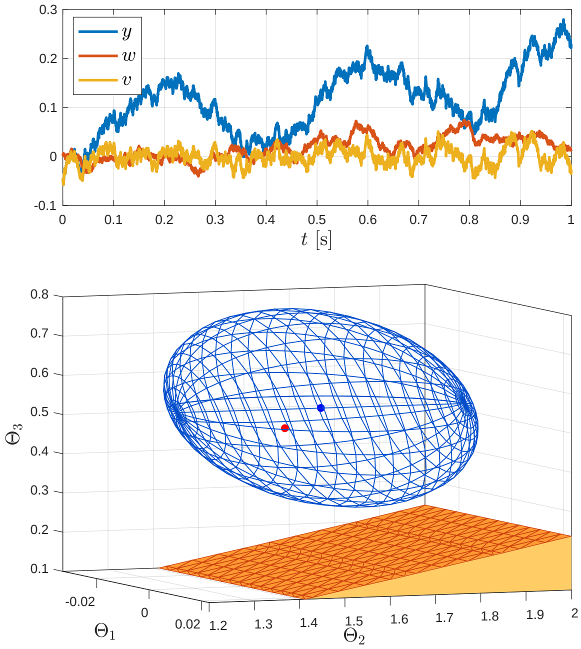

Figure 1: Scalar example. *Top:* measured output $y(\cdot)$ and noise signals $w(\cdot)$ and $v(\cdot)$. *Bottom:* representation of $\mathcal{E}$ (blue), $\mathbb{R}^3 \backslash \mathcal{C}(P, K)$ (orange), $\Theta^\star = [0\ 1.5\ 0.5]$ (red dot), and $\hat{\Theta} = [-0.0054\ 1.6106\ 0.5395]$ (blue dot).

6.1. *Scalar System with Process and Measurement Noise*

In the first example, we consider the following system:

$$\dot{x} = x + u + w, \qquad y = x + v, \tag{52}$$

which allows us to graphically illustrate the set inclusion (33) used in Theorem 1. We obtain a dataset over the interval $[0, 1]$ s by simulating (52) from $x(0) = 0$ and applying the input $u(\cdot) : t \mapsto \sin(5\pi t)$. Also, we let $\|w(\cdot)\|^2_{2,[0,1]} = \delta_w = 0.8 \times 10^{-3}$, $\|v(\cdot)\|^2_{2,[0,1]} = \delta_v = 0.3 \times 10^{-3}$. The noise signals and the measured output are reported in Fig. 1.

Then, we run Algorithm 1 with $\Lambda = -2$, $\Gamma = 2$ and $\Delta = 7.1045 \times 10^{-4}$ computed according to (47), (48), (51), with $\gamma = 0.33$ checked via (49). The resulting ellipsoid $\mathcal{E}$ as in (28) is depicted in Fig. 1, while the computed control gain K and the matrix P obtained from the LMI $(\mathcal{S})$ are

$$K = \begin{bmatrix} -29.71 & -4.87 \end{bmatrix}, \quad P = \begin{bmatrix} 1.26 & -5.34 \\ -5.34 & 53.25 \end{bmatrix} \times 10^{-3}, \tag{53}$$

and the eigenvalues of (5) are $\{-2, -5.37 \pm 4.34\mathrm{i}\}$. In Fig. 1, we also depict the values of $\Theta^\star$ and $\hat{\Theta}$ and the set $\mathbb{R}^3 \backslash \mathcal{C}(P, K)$, confirming that the property (33) holds.

6.2. *Unstable Batch Reactor with Process Noise*

We now consider the model of an unstable batch reactor derived from Green and Limebeer (1995, §2.6), which we write in the form (3) by letting $m = p = n = 2$ and:

$$\begin{aligned} A_0 &= \begin{bmatrix} -20.97 & -48.63 \\ 2.643 & 5.867 \end{bmatrix}, A_1 = \begin{bmatrix} 5.297 & -10.47 \\ -0.2764 & 6.371 \end{bmatrix} \\ B_0 &= \begin{bmatrix} -59.44 & -12.63 \\ 12.59 & 0.8696 \end{bmatrix}, B_1 = \begin{bmatrix} 0 & -3.146 \\ 5.679 & 0 \end{bmatrix}. \end{aligned} \tag{54}$$

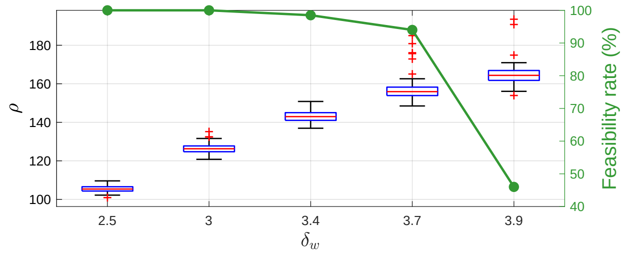

Figure 2: Batch reactor example. Each value of δ_w corresponds to 200 simulation runs. *Left axis:* box plot of ρ as defined in (42). *Right axis:* percentage of times the LMI $(\mathcal{S})$ is feasible.

We assume that the process noise $w \in \mathbb{R}^2$ affects the system via $E_0 = I_2$, $E_1 = 0_2$, while we let $v = 0$.

In this example, we run Algorithm 1 several times to highlight the effect of the noise energy on the feasibility of the LMI $(\mathcal{S})$. Each run considers a dataset over $[0, 3]$ s where $x(0) = 0$, $u(\cdot)$ is a sum of sinusoids (the same in each run), and $w(\cdot)$ is randomly generated. In particular, given five values of δ_w, we generate 200 signals uniformly extracted from the ball of radius $\sqrt{\delta_w}$ in $\mathcal{L}_2[0, 3]$, approximated via a Fourier orthonormal basis comprising the constant signal and sine/cosine functions with frequencies $2j\pi/3$, $j \in \{1, \dots, 100\}$. In all simulations, we use the filter matrices

$$\Lambda = \begin{bmatrix} 0 & -12 \\ 1 & -7 \end{bmatrix}, \qquad \Gamma = \begin{bmatrix} 0 \\ 1 \end{bmatrix}. \tag{55}$$

and let $\Delta = \gamma^2 \delta_w I_2$, with $\gamma = 0.07685$ checked via (49). The results are depicted in Fig. 2 and show that the feasibility of $(\mathcal{S})$ decreases as ρ in (42) increases.

7. Conclusion

We have proposed a data-driven approach to stabilize continuous-time LTI systems using the noisy input–output data collected from an experiment. The resulting algorithm involves a data-based LMI whose feasibility, under an interval excitation condition and a known noise energy bound, is both necessary and sufficient for quadratic stabilization. We have also related the LMI feasibility to the dataset's signal-to-noise ratio and provided tuning guidelines based on the process and measurement noise finite-horizon $\mathcal{L}_2[0, T]$ norms. Future work will extend the approach to broader classes of systems and consider datasets affected by stochastic noise or given by finite samples instead of a full continuous-time trajectory.

8. Appendix

8.1. *Derivation of the Input–Output Model*

To obtain the differential equation (1) from system (3), we perform an elimination of x as in Polderman and Willems (1998, §6.2.2). In particular, (3) can be written as

$$\mathcal{R}_x\left(\frac{\mathrm{d}}{\mathrm{d}t}\right) x = \mathcal{R}_y \begin{bmatrix} u^\top & v^\top & w^\top & y^\top \end{bmatrix}^\top, \tag{56}$$

where

$$\mathcal{R}_x(\xi) := \begin{bmatrix} \xi I_{np} - A \\ C \end{bmatrix}, \ \mathcal{R}_y := \begin{bmatrix} B & 0_{np\times p} & E & 0_{np\times p} \\ 0_{p\times m} & -I_p & 0_{p\times q} & I_p \end{bmatrix}. \tag{57}$$

Define the polynomial matrix

$$\mathcal{W}(\xi) := \begin{bmatrix} I_p & \xi I_p & \cdots & \xi^{n-1} I_p & -\mathcal{D}(\xi) \end{bmatrix}. \tag{58}$$

Then, by pre-multiplying both sides of (56) by $\mathcal{W}(\mathrm{d}/\mathrm{d}t)$ and exploiting the structure of A, B, C, E in (3), we obtain exactly (1). Note, in particular, that straightforward computations yield $\mathcal{W}(\xi)\mathcal{R}_x(\xi) = 0$ and $\mathcal{W}(\xi)\mathcal{R}_y = [\mathcal{N}(\xi)\ \mathcal{D}(\xi)\ \mathcal{N}_w(\xi)\ -\mathcal{D}(\xi)]$.

8.2. *Observer Canonical Form*

Let $\mathrm{e}_i \in \mathbb{R}^n$, $i \in \{1, \ldots, n\}$, be the i-th component of the standard basis of $\mathbb{R}^n$ and define:

$$\bar{A} := \begin{bmatrix} 0_{1\times(n-1)} & 0 \\ I_{n-1} & 0_{(n-1)\times 1} \end{bmatrix}. \tag{59}$$

We observe that A in (3) can be written as

$$A = \bar{A} \otimes I_p + A_{\mathrm{m}} C = \bar{A} \otimes I_p + A_{\mathrm{m}}(\mathrm{e}_n^\top \otimes I_p), \tag{60}$$

where $A_{\mathrm{m}} := -[A_0^\top \ \ldots \ A_{n-1}^\top]^\top$. Consider the following permutation matrix:

$$\Phi := \begin{bmatrix} I_p \otimes \mathrm{e}_1 & I_p \otimes \mathrm{e}_2 & \cdots & I_p \otimes \mathrm{e}_n \end{bmatrix} \in \mathbb{R}^{np\times np}. \tag{61}$$

Some computations show that $\Phi^\top(I_p \otimes \bar{A})\Phi = \bar{A} \otimes I_p$ and $(I_p \otimes \mathrm{e}_n^\top)\Phi = \mathrm{e}_n^\top \otimes I_p = C$, which lead to the observer canonical form:

$$\begin{aligned} A_{\mathrm{o}} &:= \Phi A \Phi^\top = I_p \otimes \bar{A} + \tilde{A}_{\mathrm{m}}(I_p \otimes \mathrm{e}_n^\top) \\ C_{\mathrm{o}} &:= C\Phi^\top = I_p \otimes \mathrm{e}_n^\top, \end{aligned} \tag{62}$$

with $\tilde{A}_{\mathrm{m}} := \Phi A_{\mathrm{m}}$.

8.3. *Proof of Lemma 1*

Since the observability indices of the pair (C, A) are uniform (see Bosso et al. (2026, As. 3)), the existence of matrices Π and H is a direct consequence of Bosso et al. (2026, Thm. 3). Then, item 1 follows from Bosso et al. (2026, Thm. 4). To prove item 2, we note from the proof of Bosso et al. (2026, Thm. 3) that

$$\Phi(A - \Pi L C)\Phi^\top = A_{\mathrm{o}} - \Phi\Pi L C_{\mathrm{o}} = I_p \otimes (\bar{A} + \lambda_{\mathrm{m}}\mathrm{e}_n^\top), \tag{63}$$

where we used the notation of Appendix 8.2 and let $\lambda_{\mathrm{m}} := -[\lambda_0 \ \ldots \ \lambda_{n-1}]^\top$. We conclude the proof by noticing that

$$\begin{aligned} A - \Pi L C &= \Phi^\top(I_p \otimes (\bar{A} + \lambda_{\mathrm{m}}\mathrm{e}_n^\top))\Phi \\ &= \bar{A} \otimes I_p + \Phi^\top(I_p \otimes \lambda_{\mathrm{m}}\mathrm{e}_n^\top)\Phi \\ &= \bar{A} \otimes I_p + \lambda_{\mathrm{m}}\mathrm{e}_n^\top \otimes I_p = \tilde{\Lambda}. \end{aligned} \tag{64}$$

□

8.4. *Proof of Lemma 2*

By Lemma 1, there exists a nonsingular matrix Φ_λ such that $\Phi_\lambda\tilde{\Lambda}\Phi_\lambda^{-1} = I_p \otimes \Lambda$. Let $\Phi_\lambda\tilde{x}(0) = \mathrm{col}(\beta_1, \ldots, \beta_p)$, with $\beta_i \in \mathbb{R}^n$, and let $V_i \in \mathbb{R}^{n\times n}$ be the solution of

$$V_i\Lambda = \Lambda V_i, \qquad V_i\Gamma = \beta_i, \tag{65}$$

which exists and is unique by Bosso et al. (2026, Appendix II) because (Λ, Γ) is controllable. Define

$$V := \begin{bmatrix} V_1^\top & \cdots & V_p^\top \end{bmatrix}^\top \in \mathbb{R}^{np\times n}, \tag{66}$$

and let $\epsilon(t) := V\chi(t)$, where $\chi(t)$ is given in (8) and satisfies $\dot{\chi}(t) = \Lambda\chi(t)$, with $\chi(0) = \Gamma$. We obtain

$$\dot{\epsilon}(t) = V\Lambda\chi(t) = \Phi_\lambda\tilde{\Lambda}\Phi_\lambda^{-1}\epsilon(t), \quad \epsilon(0) = V\Gamma = \Phi_\lambda\tilde{x}(0), \tag{67}$$

which yields

$$C\Phi_\lambda^{-1}V\chi(t) = C\Phi_\lambda^{-1}e^{\Phi_\lambda\tilde{\Lambda}\Phi_\lambda^{-1}t}\Phi_\lambda\tilde{x}(0) = Ce^{\tilde{\Lambda}t}\tilde{x}(0), \tag{68}$$

and, thus, $H_0 := C\Phi_\lambda^{-1}V$. □

8.5. *Proof of Corollary 1*

By Assumption 1, Lemma 1 ensures that the pair $(F + LH, G)$ is controllable. As a consequence, there exist $\Omega = \Omega^\top \succ 0$ and K such that

$$\Omega\left(F + L\Theta^\star \begin{bmatrix} 0_{n\times\mu} \\ I_\mu \end{bmatrix} + GK\right) + \left(F + L\Theta^\star \begin{bmatrix} 0_{n\times\mu} \\ I_\mu \end{bmatrix} + GK\right)^\top \Omega \preceq -I_\mu. \tag{69}$$

From (69), Ω and K ensure stabilization for all Θ such that

$$\Omega L(\Theta - \Theta^\star)\begin{bmatrix} 0_{n\times\mu} \\ I_\mu \end{bmatrix} + \begin{bmatrix} 0_{\mu\times n} & I_\mu \end{bmatrix}(\Theta - \Theta^\star)^\top(\Omega L)^\top \preceq \ell I_\mu, \tag{70}$$

for some $\ell \in (0, 1)$. Applying the norms, (70) holds if

$$|\Theta - \Theta^\star| \le \frac{\ell}{2|\Omega L|}. \tag{71}$$

Define

$$\rho^\star := \frac{\ell^2}{16|\Omega L|^2}. \tag{72}$$

Then, if the data satisfy Assumptions 2 and 3 with $\rho \in [0, \rho^\star]$, the bound (43) implies that (70) holds, thus from the inclusions $\boldsymbol{\mathcal{E}} \subseteq \hat{\boldsymbol{\mathcal{E}}} \subseteq \boldsymbol{C}(\Omega^{-1}, K)$ we obtain the statement by invoking Theorem 1. □

8.6. *Proof of Proposition 1*

In the following, we use $\Re(a)$ and $\Im(a)$ to denote the real and imaginary parts of a complex number $a \in \mathbb{C}$. Consider the following factorization of $\mathcal{G}_v(s)$:

$$\mathcal{G}_v(s) = \frac{\prod_{i=1}^{n_r}(s - \vartheta_i)\prod_{j=1}^{n_c}(s^2 - 2\Re(\varphi_j)s + |\varphi_j|^2)}{\prod_{k=1}^{n}(s - \varsigma_k)}, \tag{73}$$

where $n_r + 2n_c = n$ and $\vartheta_i \in \mathbb{R}$, $\Re(\varphi_j) \pm \mathrm{i}\Im(\varphi_j) \in \mathbb{C}$, and $\varsigma_k \in \mathbb{R}$, $i \in \{1, \ldots, n_r\}$, $j \in \{1, \ldots, n_c\}$, $k \in \{1, \ldots, n\}$ denote, respectively, the real zeros, the complex conjugate zero pairs, and the poles of $\mathcal{G}_v(s)$. Therefore, $\mathcal{G}_v(s)$ is the product of rational functions of the form

$$\mathcal{G}_{\mathrm{r}}(s) := \frac{s-\vartheta}{s-\varsigma}, \quad \mathcal{G}_{\mathrm{c}}(s) := \frac{s^2 - 2\Re(\varphi)s + |\varphi|^2}{(s-\varsigma_1)(s-\varsigma_2)}, \tag{74}$$

where, due to condition (50), it holds that $|\varsigma| \geq |\vartheta|$ and $\min\{|\varsigma_1|, |\varsigma_2|\} \geq |\varphi|$. Thus, for all $\omega \in \mathbb{R}$, we obtain

$$|\mathcal{G}_{\mathrm{r}}(\mathrm{i}\omega)|^2 = \frac{\vartheta^2 + \omega^2}{\varsigma^2 + \omega^2} \leq 1, \tag{75}$$

and

$$\begin{aligned} |\mathcal{G}_{\mathrm{c}}(\mathrm{i}\omega)|^2 &= \frac{(|\varphi|^2 - \omega^2)^2 + (2\Re(\varphi)\omega)^2}{(\varsigma_1^2 + \omega^2)(\varsigma_2^2 + \omega^2)} \\ &= \frac{(|\varphi|^2 + \omega^2)^2 - (2\Im(\varphi)\omega)^2}{(\varsigma_1^2 + \omega^2)(\varsigma_2^2 + \omega^2)} \\ &\leq \frac{(|\varphi|^2 + \omega^2)^2}{(\varsigma_1^2 + \omega^2)(\varsigma_2^2 + \omega^2)} \leq 1. \end{aligned} \tag{76}$$

We conclude the proof by noticing that both the numerator and the denominator of $\mathcal{G}_v(s)$ are monic and of the same degree, thus

$$\lim_{\omega\to\infty} |\mathcal{G}_v(\mathrm{i}\omega)| = 1, \tag{77}$$

and, as a consequence, $\|\mathcal{G}_v\|_\infty = \sup_{\omega\in\mathbb{R}} |\mathcal{G}_v(\mathrm{i}\omega)| = 1$. □

References

Alsalti, M., Lopez, V.G., Müller, M.A., 2025. Notes on data-driven output-feedback control of linear MIMO systems. IEEE Transactions on Automatic Control .

Antsaklis, P.J., Michel, A.N., 2006. Linear Systems. Birkhäuser, Boston, MA.

ApS, M., 2024. The MOSEK optimization toolbox for MATLAB manual. Version 10.2. URL: `https://docs.mosek.com/10.2/toolbox/index.html`.

Berberich, J., Scherer, C.W., Allgöwer, F., 2022. Combining prior knowledge and data for robust controller design. IEEE Transactions on Automatic Control 68, 4618–4633.

Bisoffi, A., De Persis, C., Tesi, P., 2022. Data-driven control via Petersen's lemma. Automatica 145, 110537.

Bosso, A., 2026. IMPACT4Mech/continuous-time_stabilization_from_noisy_data: ECC2026. URL: `https://doi.org/10.5281/zenodo.19453049`, doi:`10.5281/zenodo.19453049`.

Bosso, A., Borghesi, M., Iannelli, A., Notarstefano, G., Teel, A.R., 2025. Derivative-free data-driven control of continuous-time linear time-invariant systems. European Journal of Control , 101309.

Bosso, A., Borghesi, M., Iannelli, A., Notarstefano, G., Teel, A.R., 2026. Data-driven control of continuous-time LTI systems via non-minimal realizations. IEEE Transactions on Automatic Control .

De Persis, C., Tesi, P., 2020. Formulas for data-driven control: Stabilization, optimality, and robustness. IEEE Transactions on Automatic Control 65, 909–924.

Dörfler, F., Tesi, P., De Persis, C., 2023. On the certainty-equivalence approach to direct data-driven LQR design. IEEE Transactions on Automatic Control 68, 7989–7996.

Eising, J., Cortes, J., 2024. When sampling works in data-driven control: Informativity for stabilization in continuous time. IEEE Transactions on Automatic Control .

Green, M., Limebeer, D.J., 1995. Linear Robust Control. Prentice-Hall, Englewood Cliffs, NJ.

Hu, Z., De Persis, C., Tesi, P., 2025. Enforcing contraction via data. IEEE Transactions on Automatic Control .

Ioannou, P.A., Sun, J., 2012. Robust Adaptive Control. Dover, New York, NY.

Löfberg, J., 2004. YALMIP : A toolbox for modeling and optimization in MATLAB, in: In Proceedings of the CACSD Conference, Taipei, Taiwan.

Monshizadeh, N., De Persis, C., Tesi, P., 2025. A versatile framework for data-driven control of nonlinear systems. IEEE Transactions on Automatic Control .

Narendra, K.S., Annaswamy, A.M., 1989. Stable Adaptive Systems. Prentice-Hall, Englewood Cliffs, NJ.

Nortmann, B., Mylvaganam, T., 2023. Direct data-driven control of linear time-varying systems. IEEE Transactions on Automatic Control 68, 4888–4895.

Ohta, Y., Rapisarda, P., 2024. A sampling linear functional framework for data-driven analysis and control of continuous-time systems, in: 2024 IEEE 63rd Conference on Decision and Control, pp. 357–362.

Polderman, J.W., Willems, J.C., 1998. Introduction to Mathematical Systems Theory: A Behavioral Approach. Springer-Verlag, New York, NY.

Rapisarda, P., van Waarde, H.J., Çamlibel, M., 2023. Orthogonal polynomial bases for data-driven analysis and control of continuous-time systems. IEEE Transactions on Automatic Control 69, 4307–4319.

Seuret, A., Tarbouriech, S., 2024. Robust data-driven control design for linear systems subject to input saturation. IEEE Transactions on Automatic Control 69, 6191–6198.

van Waarde, H.J., Camlibel, M.K., Eising, J., Trentelman, H.L., 2023a. Quadratic matrix inequalities with applications to data-based control. SIAM Journal on Control and Optimization 61, 2251–2281.

van Waarde, H.J., Camlibel, M.K., Trentelman, H.L., 2025. Data-Based Linear Systems and Control Theory. Kindle Direct Publishing.

van Waarde, H.J., Eising, J., Camlibel, M.K., Trentelman, H.L., 2023b. A behavioral approach to data-driven control with noisy input–output data. IEEE Transactions on Automatic Control 69, 813–827.

van Waarde, H.J., Eising, J., Trentelman, H.L., Camlibel, M.K., 2020. Data informativity: A new perspective on data-driven analysis and control. IEEE Transactions on Automatic Control 65, 4753–4768.

Willems, J.C., Rapisarda, P., Markovsky, I., De Moor, B.L., 2005. A note on persistency of excitation. Systems & Control Letters 54, 325–329.