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AUTOMATIC CONTROL LABORATORY

Dual adaptive MPC using application-oriented set-membership identification

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MPC under model uncertainty

- **MPC:** Optimization based control, ensures state and input constraint satisfaction
- **Tube MPC:** Incorporates model uncertainty into the optimization, guarantees **robustness**
- **Adaptive MPC:** A novel technique, to reduce the model uncertainty online using measurements

Features of Adaptive MPC

- Systems with linear, time-invariant dynamics and parametric uncertainties in the state space matrices and additive disturbances

$$x_{k+1} = A(\theta)x_k + B(\theta)u_k + w_k,$$
$$\big(A(\theta), B(\theta)\big) = (A_0, B_0) + \Sigma_i (A_i, B_i)\theta_i$$

- The parameters and disturbances are assumed to be bounded $w \in W, \theta \in \Theta_0$
- Constraint satisfaction ensured using tube MPC
- Uncertain parameters are bounded by sets of non-increasing size using set-membership ID
- Identification is *passive* (**PAMPC**)

Set-membership identification

- Applicable when systems are subject to bounded noise, even with unknown properties
- Compute feasible parameters at each time step:

$$\Delta_k := \{\theta | x_{k+1} - A(\theta)x_k + B(\theta)u_k \in W\}$$

- Update parameter set such that Θ_k has predefined hyperplane directions and

$$\Theta_k \supseteq \Theta_{k-1} \cap \Delta_k, \qquad \Theta_k \coloneqq \{\theta | H_\theta \theta \le h_{\theta_k}\}$$

- Can be performed by solving linear programs

Contribution

Augment Adaptive MPC with active exploration: Dual AMPC (**DAMPC**)

Main features

- Addresses the exploration-exploitation tradeoff: informative data vs. control performance
- Enables reference tracking under uncertain input setpoints
- Preserves guarantees of constraint satisfaction and recursive feasibility

Predicted parameter set

- Using a parameter estimate $\bar{\theta}_k$, predict the next state measurement as a function of u_k

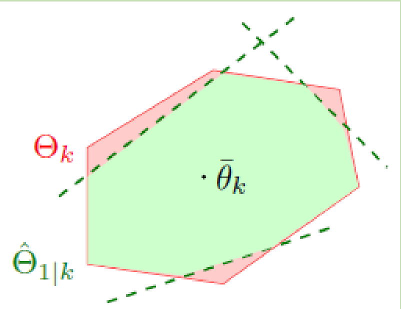
$$\hat{x}_{1|k} = A(\bar{\theta}_k)x_k + B(\bar{\theta}_k)u_k$$

- The predicted state measurement defines a feasible parameter set as

$$\hat{\Delta}_{1|k} := \{\theta | \hat{x}_{1|k} - A(\theta)x_k + B(\theta)u_k \in W\}$$

- The identification at the next time step can be approximated as

$$\hat{\Theta}_{1|k} \coloneqq \Theta_k \cap \hat{\Delta}_{1|k}$$



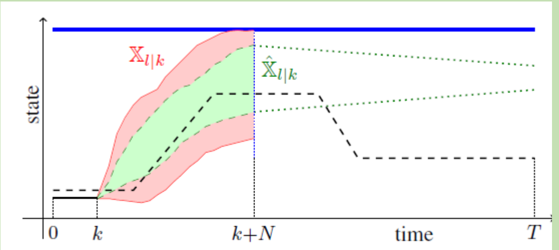
- Thus, $\hat{\Theta}_{1|k}$ **captures the effect of u_k** on identification to be performed **in the future**

Safe active exploration

- Construct state tubes using homothetic sets

$$X_{l|k} = z_{l|k} \oplus \alpha_{l|k} X_0$$

- Decouple robustness and exploration:
  - **Robust state tube**: constraint satisfaction
  - **Predicted state tube**: worst case performance as MPC cost function



Numerical Example

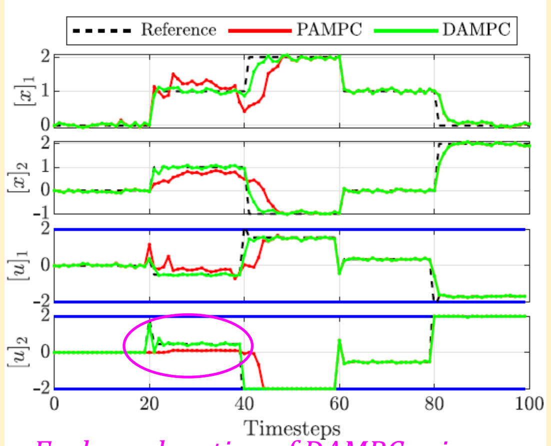
- Monte Carlo simulation study
- Compare performance of PAMPC and DAMPC*
- We consider a 2[nd] order system of the form:

$$A_0 = \begin{bmatrix} 0.85 & 0.5 \\ 0.2 & 0.7 \end{bmatrix}, B_0 = \begin{bmatrix} 1 & 0.4 \\ 0.2 & 0.6 \end{bmatrix}$$
$$A_1 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.2 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$
$$A_2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \qquad B_2 = \begin{bmatrix} 0 & 0.2 \\ 0 & 0.35 \end{bmatrix}$$

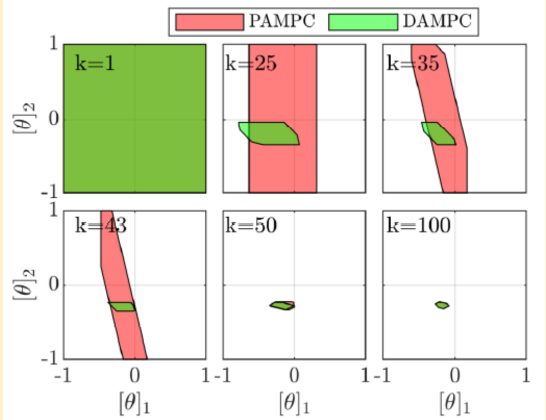
- Parameter, disturbance and constraint sets

$$||\theta||_\infty \le 1, \qquad ||w||_\infty \le 0.1,$$
$$||x||_\infty \le 3, \qquad ||u||_\infty \le 2$$

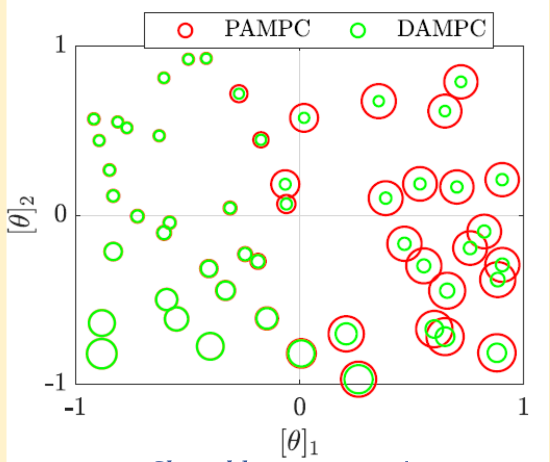
- 50 random realizations of true $\theta \in \Theta_0$
- 4 random disturbance sequences $w_k \in W$
- Finite horizon reference tracking, 100 timesteps
- Prediction horizon: 8 steps



Early exploration of DAMPC using u_2



Smaller parameter sets of DAMPC



Closed loop cost $\propto$ Area

A. Parsi, A. Iannelli and R. S. Smith, "Active exploration in adaptive model predictive control," *2020 59th IEEE Conference on Decision and Control (CDC)*

A. Parsi, A. Iannelli and R. S. Smith, "An explicit dual control approach for optimal tracking of uncertain systems," IEEE Transactions on Automatic Control (submitted), 2021

*The code is available in the public repository: https://gitlab.ethz.ch/aparsi/active-exploration-ampc-cdc-2020