

Maximum Likelihood Signal Matrix Model for Data-Driven Predictive Control

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https://youtu.be/D02wY_DtoKM

1 Signal matrix model (SMM)

Why?

Model-based control requires accurate models of system response.

Challenges: systems are increasingly complex.

Solution: moving from compact parametric models to non-parametric input-output mapping.

Novelty: a statistically optimal approach to deal with noisy data.

What?

An **implicit input-output mapping** derived directly from the signal matrix of **noise-corrupted** data.

Signal matrix: Hankel matrix of input-output trajectory data

$$U = \begin{bmatrix} u_0^d & u_1^d & \cdots & u_{M-1}^d \\ \vdots & \vdots & \ddots & \vdots \\ u_{L-1}^d & u_{L_0}^d & \cdots & u_{N-1}^d \end{bmatrix}, \quad Y = \begin{bmatrix} y_0^d & y_1^d & \cdots & y_{M-1}^d \\ \vdots & \vdots & \ddots & \vdots \\ y_{L-1}^d & y_{L_0}^d & \cdots & y_{N-1}^d \end{bmatrix}$$

Noise-free case: *Willems' fundamental lemma* (Willems, 2005)

$$\begin{bmatrix} \mathbf{u}_{ini} \\ \mathbf{y}_{ini} \\ \mathbf{u} \end{bmatrix} = \begin{bmatrix} U_p \\ Y_p \\ U_f \end{bmatrix} g, \quad \mathbf{y} = Y_f g^*(\mathbf{u}, \mathbf{u}_{ini}, \mathbf{y}_{ini}; U_f, U_p, Y_p)$$

Noisy case: *How to estimate g?* Maximum likelihood estimation

$$g^* = \arg \max_g p(\varepsilon_y, \mathbf{y}|g)$$

$$= \arg \min_{g \in \mathcal{G}} \log \det \Sigma_y(g) + \begin{bmatrix} Y_p g - \mathbf{y}_{ini} \\ 0 \end{bmatrix}^T \Sigma_y^{-1}(g) \begin{bmatrix} Y_p g - \mathbf{y}_{ini} \\ 0 \end{bmatrix}$$

↑
posterior covariance

How?

An approximate iterative QP algorithm:

$$g^{k+1} = \arg \min_{g \in \mathcal{G}} \lambda(g^k) \|g\|_2^2 + \|Y_p g - \mathbf{y}_{ini}\|_2^2, \quad \lambda(g^k) = \frac{L' \sigma_p^2}{\|g^k\|_2^2} + L \sigma^2$$

Closed-form solution:

$$g^{k+1} = \mathcal{P}(g^k) \mathbf{y}_{ini} + \mathcal{Q}(g^k) \begin{bmatrix} \mathbf{u}_{ini} \\ \mathbf{u} \end{bmatrix}$$

2 Data-Driven Predictive Control with SMM

Idea

Optimal reference tracking by receding horizon control:

$$\begin{aligned} & \underset{\mathbf{u}, \mathbf{y}}{\text{minimize}} && \sum_{k=0}^{L'-1} (\|y_k - r_{t+k}\|_Q^2 + \|u_k\|_R^2) \\ & \text{subject to} && \left(\begin{bmatrix} \mathbf{u}_{ini} \\ \mathbf{u} \end{bmatrix}, \begin{bmatrix} \mathbf{y}_{ini} \\ \mathbf{y} \end{bmatrix} \right) \text{ is a possible system trajectory } (*) \\ & && \mathbf{u} \in \mathcal{U}, \quad \mathbf{y} \in \mathcal{Y} \end{aligned}$$

Instead of a compact model, (*) can be formulated in a data-driven approach (Coulson, 2019).

$$\text{With SMM: } \mathbf{y} = Y_f g_{SMM}^*(\mathbf{u}, \mathbf{u}_{ini}, \mathbf{y}_{ini})$$

Computation

1) Approximate the signal matrix model with one iteration, warm-starting from the previous g -value \rightarrow linear constraint

$$(*) \leftrightarrow \mathbf{y} = Y_f (\mathcal{P}(g^{t-1}) \mathbf{y}_{ini} + \mathcal{Q}(g^{t-1}) \begin{bmatrix} \mathbf{u}_{ini} \\ \mathbf{u} \end{bmatrix})$$

2) Precondition by **compressing the signal matrix** \rightarrow online computations does not depend on data length

$$\begin{bmatrix} U \\ Y \end{bmatrix} \xrightarrow{\text{svd}} W S V^T, \quad \begin{bmatrix} \hat{U} \\ \hat{Y} \end{bmatrix} \triangleq W S(:, 1:2L)$$

Salient features

1) **Incorporation of online data:** online measurements added to the signal matrix **forgetting factor**

$$\begin{bmatrix} U_{t+1} \\ Y_{t+1} \end{bmatrix} = \begin{bmatrix} \gamma U_t & (u_i)_{i=t-L+1}^t \\ \gamma Y_t & (y_i)_{i=t-L+1}^t \end{bmatrix}$$

Beneficial for low SNR data and/or slowly varying systems

2) **Regularized SMM predictive control:** additional objective of reducing prediction error

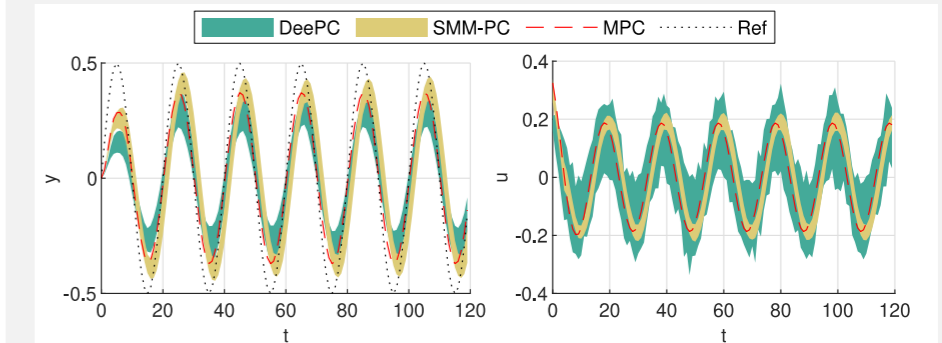
$$(\Sigma_{y_f})_{i,i} = \sigma^2 \|g\|_2^2, \quad J_{\text{Reg-SMM}} = J_{\text{ctr}} + \zeta \cdot \sigma^2 \|g^t\|_2^2$$

Tune ζ to add robustness to the controller

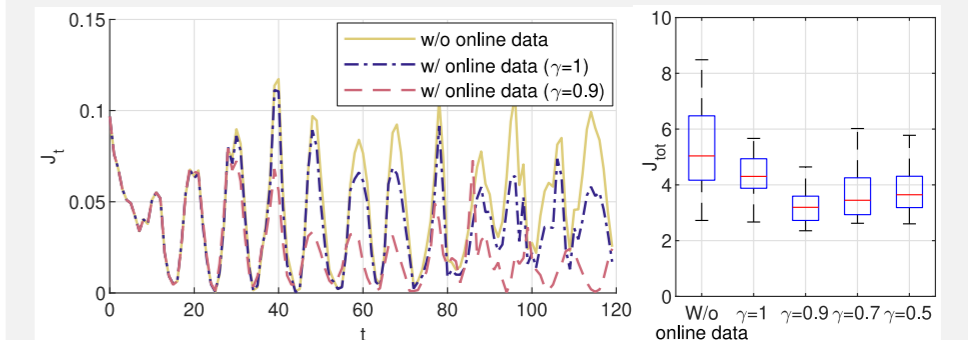
3 Results

Case study: 4th-order linear system, sinusoidal reference, no I/O constraints, known noise levels.

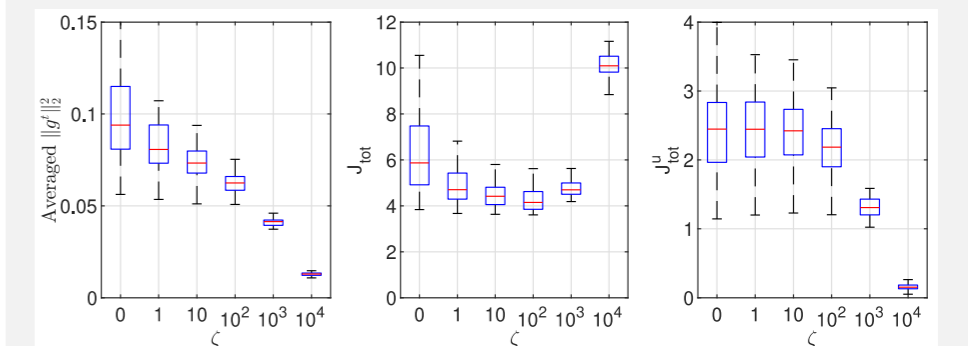
Benchmark: ideal MPC & DeePC (Coulson, 2019)



Closed-loop trajectory comparison with ideal MPC & DeePC



Online data adaptation for system with slow parameter drifts



Performance of regularized SMM predictive control

References

Mingzhou Yin, Andrea Iannelli, and Roy S. Smith. Maximum likelihood estimation in data-driven modeling and control. arXiv preprint arXiv:2011.00925, 2020.